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Zero-sum partitions of Abelian groups of order 2^n

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The following problem has been known since the 80's. Let Γ be an Abelian group of order m (denoted $|\Gamma| = m$), and let t and m_i, $1 \leq i \leq t$, be positive integers such that $\sum_{i=1}^{t} m_i = m - 1$. Determine when $\Gamma^* = \Gamma \setminus \{0\}$, the set of non-zero elements of Γ, can be partitioned into disjoint subsets S_i, $1 \leq i \leq t$, such that $|S_i| = m_i$ and $\sum_{s \in S_i} s = 0$ for every $i \in [1, t]$.

It is easy to check that $m_i \geq 2$ (for every $i \in [1, t]$) and $|I(\Gamma)| \neq 1$ are necessary conditions for the existence of such partitions, where $I(\Gamma)$ is the set of involutions of Γ. It was proved that the condition $m_i \geq 2$ is sufficient if and only if $|I(\Gamma)| \in \{0, 3\}$ (see Zeng, (2015)).

For other groups (i.e., for which $|I(\Gamma)| \neq 3$ and $|I(\Gamma)| > 1$), only the case of any group Γ with $\Gamma \cong (\mathbb{Z}_2)^n$ for some positive integer n has been analyzed completely so far, and it was shown independently by several authors that $m_i \geq 3$ is sufficient in this case. Moreover, recently Cichacz and Tuza (2021) proved that, if $|\Gamma|$ is large enough and $|I(\Gamma)| > 1$, then $m_i \geq 4$ is sufficient.

In this paper we generalize this result for every Abelian group of order 2^n. Namely, we show that the condition $m_i \geq 3$ is sufficient for Γ such that $|I(\Gamma)| > 1$ and $|\Gamma| = 2^n$, for every positive integer n. We also present some applications of this result to graph magic- and anti-magic-type labelings.

Keywords: Abelian group, zero-sum sets, irregular labeling, antimagic labeling, distance magic labeling

1 Introduction

1.1 Main problem

Let Γ be an Abelian group of order m with the operation denoted by $+$[(i)]. For convenience, we will denote $\sum_{i=1}^{k} a$ by ka, the inverse of a by $-a$, and $a + (-b)$ by $a - b$. Moreover, we will write $\sum_{a \in S} a$ for the sum of all elements in S. The identity element of Γ will be denoted by 0, and the set of non-zero elements of Γ by Γ^*. Recall that any element $\iota \in \Gamma$ of order 2 (i.e., $\iota \neq 0$ and $2\iota = 0$) is called an *involution*. We will write $I(\Gamma)$ for the set of involutions of Γ. A non-trivial finite group has an involution if and only if the order of the group is even. The fundamental theorem of finite Abelian groups states that every

[(i)] For standard notions, notations and results in finite algebra the reader is referred to the textbook by Gallian (2016). In this article we recall only the ones that are most directly related to the presentation of our work.

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finite Abelian group Γ is isomorphic to the direct product of some cyclic subgroups of prime-power order (Gallian, 2016). In other words, there exists a positive integer k, (not necessarily distinct) prime numbers $\{p_i\}_{i=1}^k$, and positive integers $\{\alpha_i\}_{i=1}^k$, such that

$$\Gamma \cong \mathbb{Z}_{p_1^{\alpha_1}} \times \mathbb{Z}_{p_2^{\alpha_2}} \times \ldots \times \mathbb{Z}_{p_k^{\alpha_k}}, \text{where} \quad n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \ldots \cdot p_k^{\alpha_k}.$$

Moreover, this factorization is unique (up to the order of terms in the direct product). Since any cyclic finite group of even order has exactly one involution, if e is the number of cyclic subgroups in the factorization of Γ whose order is even, then $|I(\Gamma)| = 2^e - 1$.

For positive integers a and b such that $a < b$ let $[a, b] = \{a, a+1, \ldots, b\}$.

Because the results presented in this paper are invariant under the isomorphism between groups ($\cong$), we only need to consider one group in every isomorphism class. Our presentation will be focused on groups of the form $\mathbb{Z}_{p_1^{\alpha_1}} \times \mathbb{Z}_{p_2^{\alpha_2}} \times \ldots \times \mathbb{Z}_{p_k^{\alpha_k}}$, with prime numbers $\{p_i\}_{i=1}^k$ and positive integers $\{\alpha_i\}_{i=1}^k$. For ease of presentation, when speaking of a subgroup of a group in which all elements have 0 on some positions (corresponding to some terms in the factorization), we will omit these zeros (under group isomorphism). For an Abelian group $\Gamma \cong U \times H$, given a pair $u \in U$ and $v \in H$, we will use the notation (u, v) (concatenation of u and v) for the corresponding element of Γ.

The sum of all elements of a group Γ is equal to the sum of its involutions and the identity element (Gallian, 2016). The following lemma is well known. The readers can consult Combe et al. (2004) for a proof.

Lemma 1.1 (Combe et al. (2004))**.** *Let Γ be an Abelian group.*

- *If $|I(\Gamma)| = 1$, then $\sum_{g\in\Gamma} g = \iota$, where ι is the involution.*
- *If $I|(\Gamma)| \neq 1$, then $\sum_{g\in\Gamma} g = 0$.*

In 1981 Tannenbaum introduced the following problem of partitioning in Abelian groups.

Problem 1.2 (Tannenbaum (1981))**.** *Let Γ be an Abelian group of order m. Let t and m_i, $i \in [1, t]$, be positive integers such that $\sum_{i=1}^t m_i = m - 1$. In other words, $\{m_i\}_{i=1}^t$ is an integer partition of $m - 1$. Let w_i, $i \in [1, t]$, be arbitrary elements of Γ (not necessarily distinct). Determine when there exists a subset partition $\{S_i\}_{i=1}^t$ of Γ^* (i.e., subsets S_i are pairwise disjoint and their union is Γ^*) such that $|S_i| = m_i$ and $\sum_{s\in S_i} s = w_i$ for every $i \in [1, t]$.*

If a respective subset partition of Γ^* exists, then we say that $\{m_i\}_{i=1}^t$ is *realizable* in Γ^* with $\{w_i\}_{i=1}^t$. In such a case, we say that $\{S_i\}_{i=1}^t$ *realizes* $\{m_i\}_{i=1}^t$ in Γ^* with $\{w_i\}_{i=1}^t$. We will not specify the sequence $\{w_i\}_{i=1}^t$ whenever it is clear from the context. Realizability of the sequence implies that $\sum_{i=1}^t w_i = \sum_{g\in\Gamma} g$. Analogously, in the context of any subset Z of Γ, we will say that $\{S_i\}_{i=1}^t$, a partition of Z into zero-sum subsets, realizes $\{m_i\}_{i=1}^t$ in Z if $|S_i| = m_i$ for every $i \in [1, t]$.

Note that we use sequences $\{m_i\}_{i=1}^t$ only for ease of presentation, whereas we interpret them as multisets. In other words, only the values present in the sequence and their multiplicities are relevant for a sequence to be realizable, not their order.

Since we will consider subsets of fixed cardinalities throughout the paper, let us present an abbreviated notation. Given a set, any of its subsets of cardinality k is called a *k-subset*.

The case most studied in the literature is the *Zero-Sum Partition (ZSP)* problem, i.e., when $w_i = 0$ for every $i \in [1, t]$. Note that, by Lemma 1.1, for an Abelian group Γ with $|I(\Gamma)| = 1$, Γ^* does not admit zero-sum partitions. Moreover, given a group Γ, if $\{m_i\}_{i=1}^t$ is realizable in Γ^* (with $w_i = 0$ for every $i \in [1,t]$), then necessarily $m_i \geq 2$ for every $i \in [1,t]$. It was proved that this condition is sufficient when $|I(\Gamma)| = 0$ (Tannenbaum, 1981; Zeng, 2015) or $|I(\Gamma)| = 3$ (Zeng, 2015). Let us generalize this situation with the following definition.

Definition 1.3. *Let Γ be a finite Abelian group of order m. We say that Γ has* x-Zero-Sum Partition Property (x-ZSPP) *if, for every positive integer t and every integer partition $\{m_i\}_{i=1}^t$ of $m-1$ (i.e., $\sum_{i=1}^t m_i = m-1$) with $m_i \geq x$ for every $i \in [1,t]$, there exists a subset partition $\{S_i\}_{i=1}^t$ of Γ^* (i.e., subsets S_i are pairwise disjoint and their union is Γ^*) with $|S_i| = m_i$ and $\sum_{s\in S_i} s = 0$ for every $i \in [1,t]$.*

The following theorem was first conjectured by Kaplan et al. (2009) (they also showed the necessity), and later proved by Zeng (2015).

Theorem 1.4 (Zeng (2015))**.** *Let Γ be a finite Abelian group of order m. Γ has 2-ZSPP if and only if $|I(\Gamma)| \in \{0,3\}$.*

The above theorem confirms, for the case of $|I(\Gamma)| = 3$, the following conjecture stated by Tannenbaum.

Conjecture 1.5 (Tannenbaum (1981))**.** *Let Γ be a finite Abelian group of order m with $|I(\Gamma)| > 1$. Let $R = \Gamma \setminus (I(\Gamma) \cup \{0\})$. For every positive integer t and every integer partition $\{m_i\}_{i=1}^t$ of $m-1$, with $m_i \geq 2$ for every $i \in [1, |R|/2]$, and $m_i \geq 3$ for every $i \in [|R|/2+1, t]$, there is a subset partition $\{S_i\}_{i=1}^t$ of Γ^* such that $|S_i| = m_i$ and $\sum_{s\in S_i} s = 0$ for every $i \in [1,t]$.*

It was shown independently by a few authors that Conjecture 1.5 is also true for $\Gamma \cong (\mathbb{Z}_2)^n$ for any integer n, with $n > 1$ (Note that in this case $I(\Gamma) = \Gamma^*$, so $R = \emptyset$).

Theorem 1.6 (Caccetta and Jia (1997); Egawa (1997); Tannenbaum (1983))**.** *Let $\Gamma \cong (\mathbb{Z}_2)^n$ for some integer n, with $n > 1$, then Γ has 3-ZSPP.*

Note that, in general, even the weaker version of Conjecture 1.5 posed by Cichacz (2018) is still open.

Conjecture 1.7 (Cichacz (2018))**.** *Let Γ be a finite Abelian group with $|I(\Gamma)| > 1$. Then Γ has 3-ZSPP.*

Recently Cichacz and Tuza (2022) proved that, if $|\Gamma|$ is large enough and $|I(\Gamma)| > 1$, then Γ has 4-ZSPP.

The main contribution of this paper is to show that Conjecture 1.7 holds for every Abelian group Γ the cardinality of which is a power of 2 and $|I(\Gamma)| > 1$. This result is a generalization of the result for $\Gamma \cong (\mathbb{Z}_2)^n$ by Caccetta and Jia (1997); Egawa (1997); Tannenbaum (1983). The structure of the paper is as follows. In Section 2, we start by establishing the result for $\Gamma \cong \mathbb{Z}_4 \times (\mathbb{Z}_2)^{n-2}$, for any integer n with $n \geq 3$. Then, in Section 3, we prove the main result that any Abelian group Γ of order 2^n, for a positive integer n, such that $|I(\Gamma)| > 1$ has 3-ZSPP. We use some methods developed by Egawa (1997); Zeng (2015). Subset partitions of $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^2\right)^*$, $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^3\right)^*$, $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^4\right)^*$, $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^5\right)^*$, $\left((\mathbb{Z}_4)^2 \times \mathbb{Z}_2\right)^*$, $\left((\mathbb{Z}_4)^3\right)^*$, $\left(\mathbb{Z}_8 \times (\mathbb{Z}_2)^2\right)^*$, and $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^2\right)^* + \left((\mathbb{Z}_2)^2\right)^*$ were analyzed by a computer program that we created, and sample zero-sum partitions that certify that the corresponding sequences are realizable are given in the annexes. In Section 4, we present some applications of our main result. In Section 5, we conclude with some final remarks and propose some conjectures.

1.2 Related work and applications

x-Zero-Sum Partition Property of groups can be applied in magic- and anti-magic-type labelings of graphs[(ii)]. Generally speaking, such a labeling of a graph $G = (V, E)$ is a mapping from only V or E, or their union $V \cup E$, to a set of labels, which most often is a set of integers or elements of a group. Then the weight of a graph element is typically the sum of labels of the adjacent or incident elements of one or both types. When the weight of all elements is required to be equal, then we speak of a magic-type labeling; when the weights should be all different, then we speak of an anti-magic-type labeling. Probably the best known problem in this area is the *anti-magic conjecture* by Ringel and Hartsfield (1994), which claims that the edges of every graph except K_2 can be labeled bijectively with integers $1, 2, \dots, |E|$ so that the weight of every vertex is unique. This conjecture is still open.

It is easy to check, by the Pigeonhole Principle, that in any simple graph G there exist two vertices of the same degree. The situation changes if we consider an edge labeling $f : E(G) \to \{1, \dots, k\}$ (Note that the labels do not have to be distinct) and calculate the so-called *weighted degree* of every vertex v as the sum of labels of all the edges incident to v. The labeling f is called *irregular* if the weighted degrees of all the vertices are unique (so it is an anti-magic-type labeling). The smallest value of k that allows some irregular labeling is called the *irregularity strength of* G and denoted by $s(G)$. The problem of finding $s(G)$ was introduced by Chartrand et al. (1988) and investigated by numerous authors (Aigner and Triesch, 1990; Amar and Togni, 1998; Kalkowski et al., 2011).

Anholcer et al. (2015) introduced the *group irregular labeling*. They label the edges with (not necessarily distinct) elements of a group Γ, and then the weighted degree of a vertex v is the sum (taken in Γ) of labels of the edges incident to v. A labeling is Γ*-irregular*, if the resulting weighted degrees are pairwise distinct. The smallest k, such that for every Abelian group Γ of order k there exists a Γ-irregular labeling of G, is called the *group irregularity strength* of G and denoted $s_g(G)$. Note that $s(G) \leq s_g(G)$ for every graph G.

Anholcer and Cichacz (2017) used 2-ZSPP of groups of odd order for bounding the group irregularity strength of disconnected graphs without a star as a connected component. Roughly speaking, the authors divide every connected component into 2-subsets and 3-subsets of vertices and partition the set of non-zero elements of the corresponding group into the same number of zero-sum 2-subsets and 3-subsets, and later use the method of augmented paths to do the labeling. They obtain that $s_g(G) \leq 2\lfloor |V(G)|/2 \rfloor + 1$.

An analogous *group irregular labeling for directed graphs*, where the weight of every vertex is the sum of the labels flowing into the vertex minus the sum of labels flowing out of the vertex, was considered by Aigner and Triesch (1994), Caccetta and Jia (1997), Cichacz and Tuza (2022), Egawa (1997), Fukuchi (1998) and Tuza (1990). It turns out that there exists a Γ-irregular labeling of a directed graph $\overrightarrow{G}$ with weakly connected components $\{\overrightarrow{G_i}\}_{i=1}^t$ if and only if there exist in Γ pairwise disjoint subsets $\{S_i\}_{i=1}^t$ such that $|S_i| = |V(\overrightarrow{G_i})|$ and $\sum_{s \in S_i} s = 0$ for every $i \in [1, t]$ (see Theorem 2.1 in Cichacz and Tuza (2022)).

Kaplan et al. (2009) and Zeng (2015) used results on zero-sum partitions of Abelian groups for another type of anti-magic labeling. They label the edges bijectively with the non-zero elements of a group Γ. The weight of every vertex is calculated as the sum (taken in Γ) of the labels of incident edges. All the weights should be different. They showed that every 2-tree[(iii)] T of order n admits a Γ-anti-magic labeling (in the

[(ii)] For standard terms and notations in graph theory, the reader is referred to the textbook by Diestel (2017). For an introduction to magic- and anti-magic-type labelings, the reader is referred to the monograph by Bača et al. (2019).

[(iii)] A 2-tree T is a rooted tree, where every vertex which is not a leaf has at least two children.

above sense) for any group Γ of order n such that $|I(\Gamma)| \in \{0, 3\}$.

Froncek (2013) defined the notion of *group distance magic labeling*. In this case, the vertices of the graph are labeled through a bijection with the elements of an Abelian group Γ. The weight of every vertex is computed as the sum (in Γ) of the labels assigned to its neighbors. If all the weights are the same, then it is a Γ-*distance magic labeling*.

Note that a group Γ of order m has a constant sum partition (of all of its elements) if and only if there exists a certain complete multipartite Γ-distance magic graph. Indeed, suppose we have a constant sum partition of Γ: $\{S_i\}_{i=1}^t$ of Γ with $|S_i| = m_i$ and $\sum_{s \in S_i} s = \nu$ for every $i \in [1, t]$ and some $\nu \in \Gamma$. Let G be a complete t-partite graph with the color classes $\{A_i\}_{i=1}^t$, where $|A_i| = m_i$ for every $i \in [1, t]$. Let us label the vertices of A_i with distinct elements of S_i for every $i \in [1, t]$, and compute the weight of every vertex as the sum (in Γ) of the labels of its neighbors. It is easy to see that all the weights are equal and thus we have a Γ-distance magic labeling. On the other hand, suppose $G = K_{m_1,\ldots,m_t}$ is a complete t-partite graph of order m with the color classes $\{A_i\}_{i=1}^t$ that is Γ-distance magic with a labeling ℓ. So $\sum_{i=1, i\neq j}^t \sum_{x \in A_i} \ell(x) = \mu$ for every $j \in [1, t]$, which implies that $\sum_{x \in A_j} \ell(x) = \nu$ for every $j \in [1, t]$, and some $\nu \in \Gamma$.

2 Realizable triples in $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^{n-2}\right)^*$

Throughout this section, let n be an integer, $n \geq 3$, and let $Y \cong \mathbb{Z}_4 \times (\mathbb{Z}_2)^{n-2}$. For a subset S of Y, let $\langle S \rangle$ denote the subgroup of Y generated by S. To simplify the notation, we will write just $\langle s_1, \ldots, s_k \rangle$ instead of $\langle \{s_1, \ldots, s_k\} \rangle$. For any subsets S and T of Y, let $S + T = \{u + v \colon u \in S, v \in T\}$. Recall that the exponent of a group Γ, denoted by $e(\Gamma)$, is defined as the least common multiple of the orders of all elements of the group. Thus every non-zero subgroup of Y has its exponent equal to either 2 or 4. The order of Y is $m = 2^n$, and we will consider integer partitions $\{m_i\}_{i=1}^t$ of $m - 1$ for some positive integer t, with $m_i \geq 3$ for every $i \in [1, t]$.

Note that, since $m_i \geq 3$ for every $i \in [1, t]$, we can modify the sequence $\{m_i\}_{i=1}^t$ by subdividing every term larger than 5 into a combination of terms 3, 4, and 5. It is easy to check that, if the non-zero elements of a group Γ can be partitioned into zero-sum subsets of cardinalities corresponding to the elements of the new sequence, then the same holds for the old sequence. So we can focus only on sequences with $m_i \in \{3, 4, 5\}$ for every $i \in [1, t]$.

Let Z be a subset of Y. Let $\mathcal{K}$ be a partition of Z into zero-sum subsets with cardinalities in $\{3, 4, 5\}$, and let $a = |\{S \in \mathcal{K} \colon |S| = 3\}|$, $b = |\{S \in \mathcal{K} \colon |S| = 4\}|$ and $c = |\{S \in \mathcal{K} \colon |S| = 5\}|$. In this situation, we say that $\mathcal{K}$ *realizes* the *triple* (a, b, c) in Z. If there exists a partition realizing (a, b, c) in Z, we say that (a, b, c) is *realizable* in Z. Note that a triple (a, b, c) can be seen as a compact representation of a sequence $\{m_i\}_{i=1}^t$ in which a elements are equal to 3, b elements are equal to 4, and c elements are equal to 5.

Let us start by recalling three useful lemmas.

Lemma 2.1 (Egawa (1997)). *Let r be a non-negative integer. Let a, b, c be non-negative integers such that $3a + 4b + 5c \geq 45r + 12$ and*

$$[(b - 1)/9] \leq (a/3) + c. \tag{1}$$

Then there exist non-negative integers $\{x_i\}_{i=1}^r$, $\{y_i\}_{i=1}^r$, $\{z_i\}_{i=1}^r$, such that $3x_i + 4y_i + 5z_i = 45$ for every $i \in [1, r]$, $\sum_{i=1}^r x_i \leq a$, $\sum_{i=1}^r y_i \leq b$, and $\sum_{i=1}^r z_i \leq c$.

Lemma 2.2 (Egawa (1997)). *Let $X \cong (\mathbb{Z}_2)^n$, for some integer n, with $n \geq 2$. If n is odd, let W denote a subgroup of X of order 2^3; if n is even, let $W = \{0\}$. Then the triple $((|X| - |W|)/3, 0, 0)$ is realizable in $X \backslash W$.*

Lemma 2.3 (Cichacz (2018)). *Let Γ be an Abelian group such that $|I(\Gamma)| > 1$. Let k be a positive even integer, with $k \geq 4$, such that k divides $|\Gamma|$. Then there exists a partition of Γ into sets $\{A_i\}_{i=1}^{|\Gamma|/k}$ such that $|A_i| = k$ and $\sum_{a \in A_i} a = 0$ for every $i \in [1, |\Gamma|/k]$.*

We will also need the following lemmas:

Lemma 2.4. *Let $\Gamma \cong H \times \mathbb{Z}_{2^{n_1}}$ be an Abelian group such that $|H| = 4n$, $|I(H)| > 1$, $n_1 \in \{1, 2\}$, and n is a positive integer. Let a, b, c be non-negative integers with $3a+4b+5c = |\Gamma|-1$ and $b \geq (2^{n_1}-1)n$. If there exists a partition $\mathcal{K}$ realizing $(a, b-(2^{n_1}-1)n, c)$ in H^*, then (a, b, c) is realizable in Γ^*.*

Proof: By Lemma 2.3, there exists a partition $\mathcal{A}$ of H into zero-sum 4-subsets. For every $K \in \mathcal{K}$, let $K_0 = \{(g, 0) : g \in K\}$, and let $\mathcal{K}_0 = \{K_0 \colon K \in \mathcal{K}\}$. For every $A \in \mathcal{A}$ and every $i \in [1, 2^{n_1}-1]$, let $A_i = \{(g, i) \colon g \in A\}$, and let $\mathcal{A}_i = \{A_i \colon A \in \mathcal{A}\}$. Observe that $\mathcal{K}_0$ realizes the triple $(a, b-(2^{n_1}-1)n, c)$ in $H^* \times \{0\}$, whereas $\bigcup_{i=1}^{2^{n_1}-1} \mathcal{A}_i$ realizes the triple $(0, (2^{n_1}-1)n, 0)$ in $H \times (\mathbb{Z}_{2^{n_1}})^*$. Thus $\mathcal{K}_0 \cup \bigcup_{i=1}^{2^{n_1}-1} \mathcal{A}_i$ realizes (a, b, c) in Γ^*. □

Lemma 2.5. *Let $W \cong (\mathbb{Z}_2)^3$ be a subgroup of Y, let P be a zero-sum 3-subset of Y, suppose that $W \cap \langle P \rangle = \{0\}$. Then $(8, 0, 0)$ is realizable in $W + P$.*

Proof: Choose v_0, v_1, v_2 to be distinct elements of Y that generate W, so $W = \langle v_0, v_1, v_2 \rangle$. Let p_0, p_1, p_2 be the elements of P. In the following, the subscripts should be interpreted modulo 3. For $k \in \{0, 1, 2\}$, let $P_k = \{p_k, v_k + p_{k+1}, v_k + p_{k+2}\}$, $S_k = \{v_k + p_k, v_{k+1} + v_{k+2} + p_{k+2}, v_k + v_{k+1} + v_{k+2} + p_{k+1}\}$. For every $l \in \{0, 1\}$, let $T_l = \{v_i + v_{i+1} + p_{i+2+l} \colon 0 \leq i \leq 2\}$. Then $\{P_k : 0 \leq k \leq 2\} \cup \{S_k : 0 \leq k \leq 2\} \cup \{T_l : 0 \leq l \leq 1\}$ realizes $(8, 0, 0)$ in $W + P$. □

Lemma 2.6. *Let $W \cong (\mathbb{Z}_2)^3$ be a subgroup of Y, let R be a zero-sum 5-subset of Y, and suppose that $W \cap \langle R \rangle = \{0\}$. Then $(0, 0, 8)$ is realizable in $W + R$.*

Proof:

Let $W = \langle v_0, v_1, v_2 \rangle$, $R = \{p_0, p_1, p_2, q, r\}$, P_k and S_k for every $0 \leq k \leq 2$, and the sets T_l for every $0 \leq l \leq 1$ be similar to the definitions in the proof of Lemma 2.5. Here also the subscripts should be interpreted modulo 3.

Additionally, let $R_k = P_k \cup \{v_k + q, v_k + r\}$ and $U_k = S_k \cup \{v_{k+1} + v_{k+2} + q, v_{k+1} + v_{k+2} + r\}$ for every $0 \leq k \leq 2$, and $V_0 = T_0 \cup \{q, r\}$, $V_1 = T_1 \cup \{v_0 + v_1 + v_2 + q, v_0 + v_1 + v_2 + r\}$. Then $\{R_k : 0 \leq k \leq 2\} \cup \{U_k : 0 \leq k \leq 2\} \cup \{V_i : 0 \leq i \leq 1\}$ realizes $(0, 0, 8)$ in $W + R$. □

We consider first the cases of Y such that $n \leq 7$

Lemma 2.7. *Let n be an integer with $3 \leq n \leq 7$. Let a, b, c be non-negative integers such that $3a + 4b + 5c = 2^n - 1$. Then (a, b, c) is realizable in Y^*.*

Proof: By Theorem 1.4, $\mathbb{Z}_4 \times \mathbb{Z}_2$ has 2-ZSPP. For $n \geq 4$, the triples (a, b, c) with $b < 2^{n-3}$ were analyzed by a computer program we created, sample realizations can be found in the annexes. Therefore, for $H = \mathbb{Z}_4 \times (\mathbb{Z}_2)^{n-3}$ and $n_1 = 1$, by Lemma 2.4, we obtain that all triples (a, b, c) are realizable in Y^*. □

Theorem 2.8. *Let n be an integer with $n \geq 3$. Let a, b, c be non-negative integers such that $3a+4b+5c = 2^n - 1$. Then (a, b, c) is realizable in Y^*.*

Proof: By Lemma 2.7, we can assume that $n \geq 8$. Taking $H = \mathbb{Z}_4 \times (\mathbb{Z}_2)^{n-3}$ and $n_1 = 1$, by Lemma 2.4, we may assume

$$b < 2^{n-3}, \tag{2}$$

and hence

$$3a + 5c > 2^{n-1}. \tag{3}$$

Let $V \cong \mathbb{Z}_4 \times (\mathbb{Z}_2)^2$ and $U \cong (\mathbb{Z}_2)^{n-4}$ be subgroups of Y such that $U \cap V = \{0\}$. If n is odd, let $W \cong (\mathbb{Z}_2)^3$ be a subgroup of U; if n is even, let $W = \{0\}$. Since $n \geq 8$, we have

$$|W| \leq 2^{n-6}. \tag{4}$$

We will show that there is $a = a_1 + a_2 + a_3$, $b = b_1 + b_2 + b_3$ and $c = c_1 + c_2 + c_3$, so that (a_1, b_1, c_1), (a_2, b_2, c_2), and (a_3, b_3, c_3) are realizable in $W + V^*$, $(U \backslash W) + V^*$, and U^*, respectively.

Let us first consider $W + V^*$. By (3), we have $3a > 2^{n-2}$ or $5c > 2^{n-2}$. Assume first that $3a > 2^{n-2}$. In this case, let $(a_1, b_1, c_1) = (5|W|, 0, 0)$. By (4), we have $a_1 < a$. By Lemma 2.7, we can partition V^* into five zero-sum 3-subsets $P_0, \cdots, P_4$. Then, for every $i \in [0, 4]$, there exists a subset partition $\mathcal{K}_i$ of $W + P_i$ that realizes $(|W|, 0, 0)$ (for n odd we apply Lemma 2.5). Hence $\mathcal{K} = \bigcup_{i=0}^{4} \mathcal{K}_i$ realizes (a_1, b_1, c_1) in $W + V^*$. Assume now that $3a \leq 2^{n-2}$, so $5c > 2^{n-2}$. In this case, we take $(a_1, b_1, c_1) = (0, 0, 3|W|)$. By (4), we have $c_1 < c$. By Lemma 2.7, we can partition V^* into three zero-sum 5-subsets R_0, R_1, R_2. Then we can see from Lemma 2.6 that, for every $i \in \{0, 1, 2\}$, there exists a subset partition $\mathcal{K}_i$ of $W + R_i$ realizing $(0, 0, |W|)$. Thus, the partition $\mathcal{K} = \bigcup_{i=0}^{2} \mathcal{K}_i$ realizes (a_1, b_1, c_1) in $W + V^*$.

Let us now consider $(U \backslash W) + V^*$ and U^*. Let $r = (|U| - |W|)/3$. Then

$$3(a - a_1) + 4(b - b_1) + 5(c - c_1) = 45r + |U^*|. \tag{5}$$

Like Egawa (1997), we also have

$$[((b - b_1) - 1)/9] < (a - a_1)/3 + (c - c_1).$$

Since (5) implies $3(a - a_1) + 4(b - b_1) + 5(c - c_1) = 45r + (2^{n-4} - 1) \geq 45r + 15$, it now follows from Lemma 2.1 that there exist non-negative integers $\{x_i, y_i, z_i\}_{i=1}^{r}$ such that

$$3x_i + 4y_i + 5z_i = 45 \tag{6}$$

for every $i \in [1, r]$, $\sum_{i=1}^{r} x_i \leq a - a_1$, $\sum_{i=1}^{r} y_i \leq b - b_1$ and $\sum_{i=1}^{r} z_i \leq c - c_1$. Let $a_2 = \sum_{i=1}^{r} x_i$, $b_2 = \sum_{i=1}^{r} y_i$, $c_2 = \sum_{i=1}^{r} z_i$, $a_3 = a - a_1 - a_2$, $b_3 = b - b_1 - b_2$, and $c_3 = c - c_1 - c_2$. By Theorem 1.6, it follows from (5) and (6) that there exists a subset partition $\mathcal{L}$ of U^* realizing (a_3, b_3, c_3). By Lemma 2.2, we can partition $U \backslash W$ into r zero-sum 3-subsets $\{S_i\}_{i=0}^{r-1}$. Since $S_i \cong ((\mathbb{Z}_2)^2)^*$ for every $i \in [0, r-1]$, the solutions for $S_i + V^*$, available in the annexes certify that, for every i, there exists a partition $\mathcal{N}_i$ realizing (x_i, y_i, z_i) in $S_i + V^*$. Then $\bigcup_{i=0}^{r-1} \mathcal{N}_i$ realizes (a_2, b_2, c_2) in $(U \backslash W) + V^*$. Hence, the partition $\mathcal{K} \cup \mathcal{L} \cup (\bigcup_{i=0}^{r-1} \mathcal{N}_i)$ realizes (a, b, c) in Y^*. □

3 Main result

In this section we will use the idea of good subsets from Zeng (2015). We call a 6-subset C of an Abelian group Γ *good* if $C = \{c, d, -c-d, -c, -d, c+d\}$ for some c and d in Γ. This idea is strongly connected with Skolem partitions of groups (Tannenbaum, 1981). Note that a good 6-subset is partitionable into three zero-sum 2-subsets as well as into two zero-sum 3-subsets. We will need the following lemma.

Lemma 3.1 (Cichacz (2018); Zeng (2015))**.** *Let Γ be a finite Abelian group such that $|I(\Gamma)| \neq 1$. Let Bij(Γ) denote the set of all bijections from Γ to itself. Then there exist $\phi, \varphi \in$Bij(Γ) (not necessarily distinct) such that $g + \phi(g) + \varphi(g) = 0$ for every $g \in \Gamma$.*

We start with some small cases.

Lemma 3.2. *Let Γ be any of the following groups: $(\mathbb{Z}_4)^2 \times \mathbb{Z}_2$, $(\mathbb{Z}_4)^3$, $\mathbb{Z}_8 \times (\mathbb{Z}_2)^2$. Let a, b, c be non-negative integers such that $3a + 4b + 5c = |\Gamma| - 1$. Then (a, b, c) is realizable in Γ^*.*

Proof: $\mathbb{Z}_4 \times \mathbb{Z}_4$ has 2-ZSPP by Theorem 1.4. For $\Gamma \cong (\mathbb{Z}_4)^2 \times \mathbb{Z}_{2^{n_1}}$, with $n_1 \in \{1, 2\}$, the triples (a, b, c) with $b < (2^{n_1} - 1)4$ were analyzed by a computer program we created, sample realizations can be found in the annexes. Therefore, for $H = (\mathbb{Z}_4)^2$ and $n_1 \in \{1, 2\}$, applying Lemma 2.4, we obtain that all triples (a, b, c) are realizable in Γ^*.

The case for $\mathbb{Z}_8 \times \mathbb{Z}_2$ is done by Theorem 1.4. For $\Gamma \cong \mathbb{Z}_8 \times (\mathbb{Z}_2)^2$, the triples (a, b, c) with $b < 4$ were analyzed by a computer program we created, sample realizations can be found in the annexes. Therefore, for $H = \mathbb{Z}_8 \times \mathbb{Z}_2$ and $n_1 = 1$, applying Lemma 2.4, we obtain that all triples (a, b, c) are realizable in Γ^*. □

Recall that the quotient group of Γ by H, for a subgroup H of Γ, is denoted by Γ/H. Now we state our main result.

Theorem 3.3. *Let Γ be such that $|I(\Gamma)| > 1$ and $|\Gamma| = 2^n$ for some integer n, with $n > 1$. Then Γ has 3-Zero-Sum Partition Property.*

The proof is by contradiction (using the method of smallest counterexample). Suppose the theorem is false and let Γ be the smallest group the order of which is a power of 2, with $|I(\Gamma)| > 1$, without 3-ZSPP.

Case 1. Suppose $\Gamma \cong \mathbb{Z}_{2^\alpha} \times H$, where $|H|$ is even and $\alpha \geq 3$. Note that for H such that $|I(H)| = 1$, we have $\Gamma \cong \mathbb{Z}_{2^\alpha} \times \mathbb{Z}_{2^\beta}$ for $\beta > 0$ and Γ has 3-ZSPP by Theorem 1.4. Thus we can assume that $|I(H)| \geq 3$. There exists a subgroup $L \cong \mathbb{Z}_{2^{\alpha-3}} \times H$ of Γ such that $|I(L)| > 1$ and $\Gamma/L \cong \mathbb{Z}_8$. Since $\mathbb{Z}_8 = \{0, 4\} \cup \{1, 2, 5, -1, -2, -5\}$, we can choose a set of coset representatives for the subgroup L in Γ, say A, such that

$$A = \{0, e\} \cup \{d, f, -d-f, -d, -f, d+f\},$$

where $2e \in L$. Since $|I(L)| > 1$, by Lemma 3.1, there exist $\phi, \varphi \in$Bij(L) such that $g + \phi(g) + \varphi(g) = 0$ for every $g \in L$. Thus

$$\Gamma^* = L^* \cup (e + L) \cup \left(\bigcup_{g \in L} \{d + g, f + \phi(g), -d - f + \varphi(g), -d - g, -f - \phi(g), d + f - \varphi(g)\} \right),$$

where the latter 6-subsets are good. Note that $B = L \cup (e + L)$ is a subgroup of Γ such that $|I(B)| \geq 3$. So, B has 3-ZSPP. Observe that $|B| = 2^\gamma$ for some positive integer $\gamma \geq 2$. Let

$$W = \left(\bigcup_{g \in L} \{d + g, f + \phi(g), -d - f + \varphi(g), -d - g, -f - \phi(g), d + f - \varphi(g)\} \right).$$

Note that $|W| = 6|L| \equiv 0 \pmod 4$. We will prove that any triple (a, b, c) such that $3a + 4b + 5c = |\Gamma| - 1$ is realizable in Γ^* and obtain a contradiction. Assume that $r_1, \dots, r_a = 3$, $r_{a+1}, \dots, r_{a+c} = 5$, and $r_{a+c+1}, \dots, r_{a+b+c} = 4$. Let l be such that $\sum_{i=1}^{l-1} r_i \leq |B^*|$ and $\sum_{i=1}^{l} r_i > |B^*|$. Let $r'_l = |B^*| - \sum_{i=1}^{l-1} r_i$ and $r''_l = r_l - r'_l$. If ($r'_l = 0$ or $r'_l \geq 3$) and ($r''_l = 0$ or $r''_l \geq 2$), then the sequence $r_1, \dots, r_{l-1}, r'_l$ is realized by a zero-sum partition $A_1, \dots, A_{l-1}, A'_l$ of B^*. Moreover, the sequence $r''_l, r_{l+1}, r_{l+2}, \dots, r_{a+b+c}$ is realized by a zero-sum partition $A''_l, A_{l+1}, A_{l+2}, \dots, A_{a+b+c}$ of W, since W is the union of good 6-subsets and $|W| = \sum_{i=l+1}^{a+b+c} r_i + r''_l$, so we are done. Hence we have to settle only the cases where $r''_l = 1$ or $r'_l = 1$ or $r'_l = 2$.

Case 1.a. $r''_l = 1$. Then l is even and r_{l+1} is odd, since $|B^*|$ is odd and $|W|$ is even (thus $|W| - r''_l$ requires at least one odd term among $r_{l+1}, \dots, r_{a+b+c}$, and all odd terms are put at the beginning of the sequence). Suppose first $r_l = 3$, then $3l - 1 = |B^*| = 2^\gamma - 1$ and so $3l = 2^\gamma$, a contradiction. Thus, suppose that $r_l = 5$. If $b > 0$, then the triple $(a, 1, l-a-1)$ is realizable in B^*, and $(0, b-1, c+a-l+1)$ is realizable in W. Assume now that $b = 0$. If $|B^*| > 7$, then, for $a \geq 2$, the triple $(a-2, 0, l-a+1)$ is realizable in B^* and $(2, 0, c+a-l-1)$ is realizable in W. For $a \leq 1$, there is $r_{l-1} = 5$, and $(a+3, 0, l-a-2)$ is realizable in B^*, and $(1, 2, c+a-l-2)$ is realizable in W. Observe that in this case, by the construction of W, the two zero-sum 4-subset in W can be split into four zero-sum 2-subsets, and we obtain a realization of $(a, 0, c)$ in Γ^*. If $|B^*| = 7$, then $a = 1$. Now $|I(B)| = 7$, so $B \cong (\mathbb{Z}_2)^3$, which implies that $\Gamma \cong (\mathbb{Z}_2)^2 \times \mathbb{Z}_8$, and this case was considered in Lemma 3.2.

Case 1.b. $r'_l = 2$. If $r_l = 3$, then we are done as in Case 1.a.. Since $|W| \equiv 0 \pmod 4$, we can assume $r_l = 5$. For $a \geq 1$, we obtain that $(a-1, 0, l-a)$ is realizable in B^*, and $(1, b, c+a-l)$ is realizable in W. For $b \geq 1$, since $|B^*| \geq 7$, we have a realization of $(1, 1, l-2)$ in B^* and a realization of $(1, b, c-l)$ in W. By the construction of W, one zero-sum 4-subset in W can be split into two zero-sum 2-subsets, and we obtain a realization of (a, b, c) in Γ^*. The only missing case is $a = b = 0$. If $|B^*| > 7$, then actually $|B^*| \geq 15$ and $r_{l-1} = r_{l-2} = 5$, thus $(4, 0, l-3)$ is realizable in B^*. Since W is the union of good 6-subsets, we have a partition of W into four zero-sum 2-subsets and $(c-l-1)$ zero-sum 5-subsets.

Assume now that $|B^*| = 7 = |I(B)|$. But then, as in Case 1.a., we have $\Gamma \cong (\mathbb{Z}_2)^2 \times \mathbb{Z}_8$, and this case was considered in Lemma 3.2.

Case 1.c. $r'_l = 1$. Then $l > 1$, and $r''_l \geq 2$ is even. Assume first that $r_l = 3$. Since $|W| \equiv 0 \pmod 3$, we have $b + c > 0$. If now $b > 0$, then there exists a realization of $(l-2, 1, 0)$ in B^* and a realization of $(a-l+2, b-1, c)$ in W. If $|B^*| > 7$, then $|B^*| \geq 15$ and $r_{l-1} = r_{l-2} = r_{l-3} = 3$. If $b = 0$ and $c = 1$, then $|\Gamma| \equiv 0 \pmod 3$, a contradiction. Therefore we can assume that $b = 0$ and $c \geq 2$, but then $(l-4, 0, 2)$ is realizable in B^*, and $(a-l+4, 0, c-2)$ is realizable in W. For the case $|B^*| = 7 = |I(B)|$ (i.e. $\Gamma \cong (\mathbb{Z}_2)^2 \times \mathbb{Z}_8$), we apply Lemma 3.2. Suppose now that $r_l = 5$. If $r_{l-1} = 5$, then we set $r'_{l-1} = 3$ and $r''_{l-1} = 2$, and re-define $r'_l := 3$ and $r''_l := r_l - 3$. The sequence $r_1, \dots, r_{l-2}, r'_{l-1}, r'_l$ is realized by a zero-sum partition $A_1, \dots, A_{l-2}, A'_{l-1}, A'_l$ in B^*, and the sequence $r''_{l-1}, r''_l, r_{l+1}, \dots, r_{a+b+c}$ is realized by a zero-sum partition $A''_{l-1}, A''_l, A_{l+1}, \dots, A_{a+b+c}$ of W. Thus $r_{l-1} = 3$. If $b > 0$, then we have a realization of $(a-1, 1, 0)$ in B^* and a realization of $(1, b-1, c)$ in W. If $|B^*| > 7$, then $|B^*| \geq 15$ and

$r_{l-1} = r_{l-2} = r_{l-3} = 3$. Since $b = 0$ and $c \geq 2$, we obtain that $(a-3, 0, 2)$ is realizable in B^*, and $(3, 0, c-2)$ is realizable in W. The case $|B^*| = 7 = |I(B)|$ implies that $\Gamma \cong (\mathbb{Z}_2)^2 \times \mathbb{Z}_8$ and we apply Lemma 3.2.

Case 2. Suppose $\Gamma \cong \mathbb{Z}_4 \times \mathbb{Z}_4 \times H$. By Theorem 1.4 and Lemma 3.2, we can assume that $|H| \geq 8$ or $H \cong \mathbb{Z}_2 \times \mathbb{Z}_2$. Moreover, by Case 1., we can assume that $e(\Gamma) = 4$, which implies that we can assume that $|I(H)| > 1$. Then there is a subgroup $L \cong H$ of Γ such that $\Gamma/L \cong \mathbb{Z}_4 \times \mathbb{Z}_4$. Because $\mathbb{Z}_4 \times \mathbb{Z}_4 = \{(0,0),(0,2),(2,0),(2,2)\} \cup \{(0,1),(1,2),(3,1),(0,3),(3,2),(1,3)\} \cup \{(1,0),(1,1),(2,3),(3,0),(3,3),(2,1)\}$, we can choose a set of coset representatives A for the subgroup L in Γ, such that

$$A = \{0, e_1, e_2, e_1+e_2\} \cup \left(\bigcup_{i=1,2} \{d_i, f_i, -d_i - f_i, -d_i, -f_i, d_i + f_i\} \right),$$

where $2e_1, 2e_2 \in L$. Since $|I(L)| > 1$, by Lemma 3.1, there exist $\phi, \varphi \in$Bij(L) such that $g + \phi(g) + \varphi(g) = 0$ for every $g \in L$. Thus

$$\begin{aligned} \Gamma^* = \quad & L^* \cup (e_1 + L) \cup (e_2 + L) \cup (e_1 + e_2 + L) \cup \\ & \left(\bigcup_{\substack{i=1,2 \\ g \in L}} \{d_i + g, f_i + \phi(g), -d_i - f_i + \varphi(g), -d_i - g, -f_i - \phi(g), d_i + f_i - \varphi(g)\} \right), \end{aligned}$$

where the latter 6-subsets are good. Note that $L \cup (e_1+L) \cup (e_2+L) \cup (e_1+e_2+L)$ is a subgroup of Γ. So, $L \cup (e_1+L) \cup (e_2+L) \cup (e_1+e_2+L)$ has 3-ZSPP. Note that for $B = L \cup (e_1+L) \cup (e_2+L) \cup (e_1+e_2+L)$ there is $B \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times H$ and $|I(B)| \geq 15$. Now, a similar reasoning to that applied in Case 1. leads to obtaining that Γ has 3-ZSPP, a contradiction.

Case 3. $\Gamma \cong \mathbb{Z}_4 \times (\mathbb{Z}_2)^{n-2}$. In this case we are done by Theorem 2.8.

Since the case $\Gamma \cong (\mathbb{Z}_2)^n$ has been solved by Theorem 1.6 this completes the proof of the theorem. □

4 Some applications

In this section we present some applications of Theorem 3.3 to the problems mentioned in Subsection 1.2.

4.1 Group irregular labeling for directed graphs

Let $\overrightarrow{G}$ be a directed graph. Recall that, if there exists a mapping $\psi\colon E(\overrightarrow{G}) \to \Gamma$ such that the mapping $\varphi_\psi\colon V(\overrightarrow{G}) \to \Gamma$ defined by

$$\varphi_\psi(x) = \sum_{y \in N^-(x)} \psi(yx) - \sum_{y \in N^+(x)} \psi(xy), \quad (x \in V(\overrightarrow{G}))$$

is injective, then we say that ψ is a Γ-irregular labeling of $\overrightarrow{G}$. By Theorem 3.3 we obtain the following:

Corollary 4.1. *Let $\overrightarrow{G}$ be a directed graph of order n with no component of order less than 3, and let Γ be a finite Abelian group with more than one involution such that $|\Gamma| = 2^m$ for some m, $|\Gamma| > n$ and $|\Gamma| \notin \{n+2, n+3\}$. Then there exists a Γ-irregular labeling of $\overrightarrow{G}$.*

Proof: Let $\{\overrightarrow{G_i}\}_{i=1}^t$ be the weakly connected components of $\overrightarrow{G}$. Recall that there exists a Γ-irregular labeling of a directed graph $\overrightarrow{G}$ with weakly connected components $\{\overrightarrow{G_i}\}_{i=1}^t$ if and only if there exist in Γ pairwise disjoint subsets $\{S_i\}_{i=1}^t$ such that $|S_i| = |V(\overrightarrow{G_i})|$ and $\sum_{s\in S_i} s = 0$ for every $i \in [1,t]$ (see Theorem 2.1 in Cichacz and Tuza (2022)).

If $|\Gamma| = n+1$, then we can take the integer partition $\{m_i\}_{i=1}^t$ of n with $m_i = |V(\overrightarrow{G}_i)|$ for every $i \in [1,t]$, and we get the result by Theorem 3.3.

Assume now that $|\Gamma| \geq n+4$. Let $m_i = |V(\overrightarrow{G}_i)|$ for every $i \in [1,t]$, and let $m_{t+1} = |\Gamma^*| - n$. Since $m_{t+1} \geq 3$, using the sequence $\{m_i\}_{i=1}^{t+1}$, by Theorem 3.3, again, we get the result. □

4.2 Realizable triples in Γ-anti-magic labeling

Recall that, given an Abelian group Γ, a Γ-anti-magic labeling of a graph G is a bijection $f\colon E(G) \to \Gamma^*$ such that the weight of every vertex (i.e. the sum of labels of incident edges) is unique. Kaplan et al. (2009) showed that, if Γ has a unique involution, then every tree on n vertices is not Γ-anti-magic. They conjectured that a tree with $|\Gamma|$ vertices admits a Γ-anti-magic labeling if and only if Γ is not a group with a unique involution. Using the same method as Kaplan et al. (2009) for 2-trees, by Theorem 3.3, we obtain the following:

Corollary 4.2. *Every* 3*-tree*[iv] T *of order* 2^n *admits a* Γ*-anti-magic labeling if and only if* $\Gamma \not\cong \mathbb{Z}_{2^n}$.

Proof: The necessity of the condition follows directly from the work of Kaplan et al. (2009). For the sufficiency, note that the only group of order 2^n with a unique involution is the cyclic group $\mathbb{Z}_{2^n}$. Thus we can assume that $\Gamma \not\cong \mathbb{Z}_{2^n}$. Let $\{v_i\}_{i=1}^t$ be the vertices of T which are not leaves. Let us denote their corresponding numbers of children by $\{m_i\}_{i=1}^t$. Thus $\sum_{i=1}^t m_i = 2^n - 1$. Since T is a 3-tree, we have that $m_i \geq 3$ for every $i \in [1,t]$. By Theorem 3.3, Γ has 3-ZSPP, and there exists a zero-sum partition $\{S_i\}_{i=1}^t$ of Γ^*. For all $i \in [1,t]$, we label the edges of the set $E_i = \{v_i w\colon w \text{ is a child of } v_i\}$ by the elements of S_i. The edges of T are labeled bijectively with the non-zero elements of Γ, and the sum of the labels in every E_i is 0. Since every vertex of T, except the root (whose weight is 0), has a unique parent, it easily follows that the weights are pairwise distinct. □

4.3 Group distance magic labeling

Recall that in group distance magic labeling, the vertices of the graph are labeled through a bijection with the elements of an Abelian group Γ. The weight of every vertex is computed as the sum (in Γ) of the labels assigned to its neighbors. If all the weights are the same, then it is a Γ-*distance magic labeling*.

Let $G = K_{m_1,\ldots,m_t}$ be a complete t-partite graph of order m. Let now $1 \leq m_1 \leq m_2 \leq \ldots \leq m_t$. Using some constant-sum partition properties of every finite Abelian group Γ of order m, it was shown that:

1. For $t=2$, if $m_1 + m_2 \not\equiv 2 \pmod 4$ then the graph G is Γ-distance magic (Cichacz, 2014).

2. For $t=3$, if ($m_2 > 1$ and $m_1+m_2+m_3 \neq 2^p$ for any positive integer p) or ($m_1 \neq 2$ and $m_2 > 2$), then the graph G admits a Γ-distance magic labeling (Cichacz, 2017).

[iv] A 3-tree T is a rooted tree, where every vertex which is not a leaf has at least three children.

3. If $m_1 = m_2 = \ldots = m_t > 2$ and $|I(\Gamma)| \neq 1$, then the graph G admits a Γ-distance magic labeling (Cichacz, 2018).

4. If $m_1 \geq 3$ and $m_t \geq \frac{1}{2}(m + \sqrt{2m+1}) - 1$, and $|I(\Gamma)| \neq 1$, then G admits a Γ-distance magic labeling (Cichacz and Tuza, 2022).

5. If $m_1 \geq 4$, m is large enough, and $|I(\Gamma)| \neq 1$, then G admits a Γ-distance magic labeling (Cichacz and Tuza, 2022),

6. If $m_2 \geq 2$, t is odd, and $\Gamma \cong \mathbb{Z}_n$, then G admits a Γ-distance magic labeling (Freyberg, 2020).

Our main result allows us to prove the following corollary.

Corollary 4.3. *Let $G = K_{m_1,m_2,\ldots,m_t}$ be a complete t-partite graph such that $2^n = m_1 + m_2 + \ldots + m_t$ and $m_i \geq 3$ for every $1 \leq i \leq t$. Let $\Gamma \ncong \mathbb{Z}_{2^n}$ be an Abelian group of order 2^n, then the graph G admits a Γ-distance magic labeling.*

Proof: Since $\Gamma \ncong \mathbb{Z}_{2^n}$, and the required cartinalities are greater than or equal 3, by Theorem 3.3, there exists a zero-sum partition $\{A'_i\}_{i=1}^t$ of Γ^* such that $|A'_t| = m_t - 1$ and $|A'_i| = m_i$ for every $i \in [1, t-1]$. Let $A_t = A'_t \cup \{0\}$ and $A_i = A'_i$ for every $i \in [1, t-1]$. For every $i \in [1, t]$, we can label the vertices of V_i, where V_i is the color class of cardinality m_i, using the elements of the set A_i, thus obtaining the required labeling. □

5 Final remarks

Our main result confirms Conjecture 1.7, that any finite Abelian group with more than one involution has 3-Zero-Sum Partition Property for the case of groups of order 2^n. The other cases of groups of even order with more than one involution are still open (recall that the case of groups of odd order is completely solved (Tannenbaum, 1981; Zeng, 2015)). We believe that, using similar techniques (with some more effort) as in the proof of Theorem 3.3, Conjecture 1.7 could be confirmed for other groups of even order with more than one involution. The problem was partially solved in a very recent work by Müyesser and Pokrovskiy (2022), where the authors showed that, if $|\Gamma|$ is large enough and $|I(\Gamma)| > 1$, then Γ has 3-ZSPP.

On the side of applications, Corollary 4.3 covers all groups of order 2^n, except for $\mathbb{Z}_{2^n}$. It would be interesting to see what happens in this case. We know the following sufficient and necessary condition for $\mathbb{Z}_m$-distance magic labeling of a complete t-partite graph only for t or m odd.

Theorem 5.1 (Cichacz (2017); Freyberg (2020))**.** *Let $G = K_{m_1,m_2,\ldots,m_t}$ be a complete t-partite graph of order m such that $1 \leq m_1 \leq m_2 \leq \ldots \leq m_t$ and m or t is odd. The graph G admits a $\mathbb{Z}_m$-distance magic labeling if and only if $m_2 \geq 2$.*

For m and t even, we only know the following.

Theorem 5.2 (Cichacz (2014, 2018))**.** *Let $G = K_{m_1,m_2,\ldots,m_t}$ be a complete t-partite graph of order m such that $1 \leq m_1 \leq m_2 \leq \ldots \leq m_t$ and m and t are even. If $t = 2$, the graph G admits a $\mathbb{Z}_m$-distance magic labeling if and only if $m \not\equiv 2 \pmod 4$. If $2 \leq m_1 = m_2 = \ldots = m_t$ then the graph G admits a $\mathbb{Z}_m$-distance magic labeling if and only if m_1 is even.*

Let $m = 2^n(2k+1)$, $t = 2^{n'}(2k'+1)$ for $n, n', k, k' \in \mathbb{N}$, $n \geq 1$ and $n' \geq n$. Note that, on the one hand, for every $a \in \mathbb{Z}_m$, we have $ta \not\equiv m/2 \pmod{m}$. On the other hand, if the graph $G = K_{m_1,m_2,\ldots,m_t}$ is $\mathbb{Z}_m$-distance magic, with the magic constant $\mu \in \mathbb{Z}_m$, then $t\mu = (t-1)\sum_{g\in\Gamma} g \equiv m/2 \pmod{m}$ - a contradiction. Observe that for $m = 2^n$ and $t = 2^{n'}(2k'+1)$ such that $m = \sum_{i=1}^{t} m_i$, with $m_i \geq 2$ for every $i \in [1,t]$, there is $n' < n$. Thus we state the following conjecture:

Conjecture 5.3. *Let $G = K_{m_1,m_2,\ldots,m_t}$ be a complete t-partite graph such that $2^n = \sum_{i=1}^{t} m_i$ and $m_i \geq 2$ for every $i \in [1,t]$. Then G admits a $\mathbb{Z}_{2^n}$-distance magic labeling.*

We also state a similar conjecture in the language of Problem 1.2 of partitions of the group $\mathbb{Z}_{2^n}$.

Conjecture 5.4. *Let $\mathbb{Z}_m$ be a group of order $m = 2^n$. Let t and m_i, $i \in [1,t]$, be positive integers such that $\sum_{i=1}^{t} m_i = m-1$, $m_1 \geq 1$, $m_i \geq 2$ for every $i \in [2,t]$. Then there exists $\mu \in \mathbb{Z}_m$ such that the elements of $\mathbb{Z}_m^*$ can be partitioned into disjoint subsets S_i, $i \in [1,t]$, such that $|S_i| = m_i$ and $\sum_{s\in S_i} s = \mu$ for every $i \in [1,t]$.*

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Throughout the annexes, the notation "`a*3  b*4  c*5`" refers to a sets of size 3, b sets of size 4, and c sets of size 5. In each partition, we present one set per line. Each tuple represents one element of the partitioned set, with the elements of the tuple following the order of the direct product.

A Zero sum partitions of $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^2\right)^*$ (with $b \leq 1$)

```
[4, 2, 2]
[5, 0, 0]
[
[[0, 0, 1],[1, 0, 0],[3, 0, 1]],
[[0, 1, 0],[1, 0, 1],[3, 1, 1]],
[[2, 1, 0],[3, 0, 0],[3, 1, 0]],
[[0, 1, 1],[2, 0, 0],[2, 1, 1]],
[[1, 1, 0],[1, 1, 1],[2, 0, 1]]
]
A partition for sets of sizes:  5*3  0*4  0*5
```

```
[4, 2, 2]
[0, 0, 3]
[
[[0, 0, 1],[0, 1, 0],[0, 1, 1],[1, 0, 0],[3, 0, 0]],
[[1, 1, 0],[1, 1, 1],[2, 0, 0],[2, 1, 0],[2, 1, 1]],
[[1, 0, 1],[2, 0, 1],[3, 0, 1],[3, 1, 0],[3, 1, 1]]
]
A partition for sets of sizes:  0*3  0*4  3*5
```

B Zero sum partitions of $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^3\right)^*$ (with $b \leq 3$)

```
[4, 2, 2, 2]
[7, 0, 2]
[
[[0, 0, 0, 1],[0, 0, 1, 0],[0, 0, 1, 1]],
[[0, 1, 0, 0],[1, 0, 0, 0],[3, 1, 0, 0]],
[[0, 1, 1, 1],[1, 0, 0, 1],[3, 1, 1, 0]],
[[1, 0, 1, 0],[1, 0, 1, 1],[2, 0, 0, 1]],
[[1, 1, 0, 1],[1, 1, 1, 0],[2, 0, 1, 1]],
[[2, 0, 1, 0],[3, 0, 0, 0],[3, 0, 1, 0]],
[[2, 1, 0, 0],[3, 0, 0, 1],[3, 1, 0, 1]],
[[1, 1, 0, 0],[1, 1, 1, 1],[2, 0, 0, 0],[2, 1, 0, 1],[2, 1, 1, 0]],
[[0, 1, 0, 1],[0, 1, 1, 0],[2, 1, 1, 1],[3, 0, 1, 1],[3, 1, 1, 1]]
]
A partition for sets of sizes:  7*3  0*4  2*5
```

```
[4, 2, 2, 2]
[2, 0, 5]
[
[[0, 0, 0, 1],[0, 0, 1, 0],[0, 0, 1, 1]],
[[0, 1, 0, 0],[1, 0, 0, 0],[3, 1, 0, 0]],
[[0, 1, 1, 1],[1, 0, 0, 1],[1, 0, 1, 0],[1, 0, 1, 1],[1, 1, 1, 1]],
[[1, 1, 0, 0],[1, 1, 0, 1],[1, 1, 1, 0],[2, 0, 0, 0],[3, 1, 1, 1]],
[[2, 0, 0, 1],[2, 0, 1, 0],[2, 1, 0, 0],[3, 0, 0, 1],[3, 1, 1, 0]],
[[2, 0, 1, 1],[2, 1, 1, 0],[2, 1, 1, 1],[3, 0, 0, 0],[3, 0, 1, 0]],
[[0, 1, 0, 1],[0, 1, 1, 0],[2, 1, 0, 1],[3, 0, 1, 1],[3, 1, 0, 1]]
]
A partition for sets of sizes:  2*3  0*4  5*5
```

C Zero sum partitions of $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^4\right)^*$ (with $b \leq 7$)

```
[4, 2, 2, 2, 2]
[21, 0, 0]
[
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 0],[0, 0, 0, 1, 1]],
[[0, 0, 1, 0, 0],[0, 1, 0, 0, 0],[0, 1, 1, 0, 0]],
[[0, 0, 1, 1, 1],[0, 1, 0, 0, 1],[0, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0],[1, 0, 0, 0, 0],[3, 1, 0, 1, 0]],
[[0, 1, 1, 0, 1],[1, 0, 0, 0, 1],[3, 1, 1, 0, 0]],
[[0, 1, 0, 1, 1],[1, 0, 0, 1, 0],[3, 1, 0, 0, 1]],
[[1, 0, 0, 1, 1],[1, 0, 1, 0, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[1, 1, 0, 0, 1],[1, 1, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 1, 1, 0, 0],[1, 1, 1, 1, 0],[2, 0, 0, 1, 0]],
[[1, 1, 0, 0, 0],[1, 1, 1, 0, 1],[2, 0, 1, 0, 1]],
[[1, 1, 0, 1, 1],[1, 1, 1, 1, 1],[2, 0, 1, 0, 0]],
[[2, 0, 1, 1, 0],[3, 0, 0, 0, 0],[3, 0, 1, 1, 0]],
[[2, 1, 0, 0, 0],[3, 0, 1, 0, 1],[3, 1, 1, 0, 1]],
[[2, 1, 1, 0, 0],[3, 0, 1, 0, 0],[3, 1, 0, 0, 0]],
[[2, 1, 0, 0, 1],[3, 0, 1, 1, 1],[3, 1, 1, 1, 0]],
[[2, 1, 1, 0, 1],[3, 0, 0, 1, 0],[3, 1, 1, 1, 1]],
[[0, 0, 1, 0, 1],[2, 1, 0, 1, 1],[2, 1, 1, 1, 0]],
[[0, 0, 1, 1, 0],[1, 0, 1, 0, 1],[3, 0, 0, 1, 1]],
[[0, 1, 1, 1, 1],[2, 0, 0, 0, 0],[2, 1, 1, 1, 1]],
[[2, 1, 0, 1, 0],[3, 0, 0, 0, 1],[3, 1, 0, 1, 1]]
]
A partition for sets of sizes: 21*3  0*4  0*5
```

```
[4, 2, 2, 2, 2]
[18, 1, 1]
[
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 0],[0, 0, 0, 1, 1]],
[[0, 0, 1, 0, 0],[0, 1, 0, 0, 0],[0, 1, 1, 0, 0]],
[[0, 0, 1, 1, 1],[0, 1, 0, 0, 1],[0, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0],[1, 0, 0, 0, 0],[3, 1, 0, 1, 0]],
[[0, 1, 1, 0, 1],[1, 0, 0, 0, 1],[3, 1, 1, 0, 0]],
[[0, 1, 0, 1, 1],[1, 0, 0, 1, 0],[3, 1, 0, 0, 1]],
[[1, 0, 0, 1, 1],[1, 0, 1, 0, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[1, 1, 0, 0, 1],[1, 1, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 1, 1, 0, 0],[1, 1, 1, 1, 0],[2, 0, 0, 1, 0]],
[[1, 1, 0, 0, 0],[1, 1, 1, 0, 1],[2, 0, 1, 0, 1]],
[[1, 1, 0, 1, 1],[1, 1, 1, 1, 1],[2, 0, 1, 0, 0]],
[[2, 0, 1, 1, 0],[3, 0, 0, 0, 0],[3, 0, 1, 1, 0]],
[[2, 1, 0, 0, 0],[3, 0, 0, 1, 1],[3, 1, 0, 1, 1]],
[[2, 1, 0, 1, 1],[3, 0, 1, 0, 0],[3, 1, 1, 1, 1]],
[[2, 1, 1, 1, 1],[3, 0, 0, 0, 1],[3, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1],[2, 1, 0, 0, 1],[2, 1, 1, 0, 0]],
[[2, 1, 0, 1, 0],[3, 0, 1, 1, 1],[3, 1, 1, 0, 1]],
[[1, 0, 1, 0, 1],[2, 0, 0, 0, 0],[2, 1, 1, 0, 1],[3, 1, 0, 0, 0]],
[[0, 0, 1, 1, 0],[0, 1, 1, 1, 1],[2, 1, 1, 1, 0],[3, 0, 0, 1, 0],[3, 0, 1, 0, 1]]
]
A partition for sets of sizes: 18*3  1*4  1*5
```

```
[4, 2, 2, 2, 2]
[16, 0, 3]
[
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 0],[0, 0, 0, 1, 1]],
[[0, 0, 1, 0, 0],[0, 1, 0, 0, 0],[0, 1, 1, 0, 0]],
[[0, 0, 1, 1, 1],[0, 1, 0, 0, 1],[0, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0],[1, 0, 0, 0, 0],[3, 1, 0, 1, 0]],
[[0, 1, 1, 0, 1],[1, 0, 0, 0, 1],[3, 1, 1, 0, 0]],
[[0, 1, 0, 1, 1],[1, 0, 0, 1, 0],[3, 1, 0, 0, 1]],
[[1, 0, 0, 1, 1],[1, 0, 1, 0, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[1, 1, 0, 0, 1],[1, 1, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 1, 1, 0, 0],[1, 1, 1, 1, 0],[2, 0, 0, 1, 0]],
[[1, 1, 0, 0, 0],[1, 1, 1, 0, 1],[2, 0, 1, 0, 1]],
[[1, 1, 0, 1, 1],[1, 1, 1, 1, 1],[2, 0, 1, 0, 0]],
[[2, 0, 1, 1, 0],[3, 0, 0, 0, 0],[3, 0, 1, 1, 0]],
[[2, 1, 0, 0, 0],[3, 0, 0, 1, 1],[3, 1, 0, 1, 1]],
[[2, 1, 0, 1, 1],[3, 0, 1, 0, 0],[3, 1, 1, 1, 1]],
[[2, 1, 1, 1, 1],[3, 0, 0, 0, 1],[3, 1, 1, 1, 0]],
[[2, 1, 0, 0, 1],[2, 1, 1, 0, 0],[2, 1, 1, 0, 1],[3, 0, 1, 0, 1],[3, 1, 1, 0, 1]],
[[0, 0, 1, 0, 1],[0, 1, 1, 1, 1],[2, 0, 0, 0, 0],[3, 0, 0, 1, 0],[3, 1, 0, 0, 0]],
[[0, 0, 1, 1, 0],[1, 0, 1, 0, 1],[2, 1, 0, 1, 0],[2, 1, 1, 1, 0],[3, 0, 1, 1, 1]]
]
A partition for sets of sizes: 16*3  0*4  3*5
```

```
[4, 2, 2, 2, 2]
[13, 1, 4]
[
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 0],[0, 0, 0, 1, 1]],
[[0, 0, 1, 0, 0],[0, 1, 0, 0, 0],[0, 1, 1, 0, 0]],
[[0, 0, 1, 1, 1],[0, 1, 0, 0, 1],[0, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0],[1, 0, 0, 0, 0],[3, 1, 0, 1, 0]],
[[0, 1, 1, 0, 1],[1, 0, 0, 0, 1],[3, 1, 1, 0, 0]],
[[0, 1, 0, 1, 1],[1, 0, 0, 1, 0],[3, 1, 0, 0, 1]],
[[1, 0, 0, 1, 1],[1, 0, 1, 0, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[1, 1, 0, 0, 1],[1, 1, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 1, 1, 0, 0],[1, 1, 1, 1, 0],[2, 0, 0, 1, 0]],
[[1, 1, 0, 0, 0],[1, 1, 1, 0, 1],[2, 0, 1, 0, 1]],
[[1, 1, 0, 1, 1],[1, 1, 1, 1, 1],[2, 0, 1, 0, 0]],
[[2, 0, 1, 1, 0],[3, 0, 0, 0, 0],[3, 0, 1, 1, 0]],
[[2, 1, 0, 0, 0],[2, 1, 0, 0, 1],[2, 1, 0, 1, 0],[2, 1, 0, 1, 1]],
[[2, 1, 1, 0, 0],[2, 1, 1, 0, 1],[2, 1, 1, 1, 0],[3, 0, 0, 0, 1],[3, 1, 1, 1, 0]],
[[1, 0, 1, 0, 1],[2, 1, 1, 1, 1],[3, 0, 0, 1, 1],[3, 0, 1, 0, 0],[3, 1, 1, 0, 1]],
[[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[2, 0, 0, 0, 0],[3, 1, 0, 0, 0],[3, 1, 0, 1, 1]],
[[0, 1, 1, 1, 1],[3, 0, 0, 1, 0],[3, 0, 1, 0, 1],[3, 0, 1, 1, 1],[3, 1, 1, 1, 1]]
]
A partition for sets of sizes: 13*3  1*4  4*5
```

```
[4, 2, 2, 2, 2]
[11, 0, 6]
[
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 0],[0, 0, 0, 1, 1]],
[[0, 0, 1, 0, 0],[0, 1, 0, 0, 0],[0, 1, 1, 0, 0]],
[[0, 0, 1, 1, 1],[0, 1, 0, 0, 1],[0, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0],[1, 0, 0, 0, 0],[3, 1, 0, 1, 0]],
[[0, 1, 1, 0, 1],[1, 0, 0, 0, 1],[3, 1, 1, 0, 0]],
[[0, 1, 0, 1, 1],[1, 0, 0, 1, 0],[3, 1, 0, 0, 1]],
[[1, 0, 0, 1, 1],[1, 0, 1, 0, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[1, 1, 0, 0, 1],[1, 1, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 1, 1, 0, 0],[1, 1, 1, 1, 0],[2, 0, 0, 1, 0]],
[[1, 1, 0, 0, 0],[1, 1, 1, 0, 1],[2, 0, 1, 0, 1]],
[[2, 0, 0, 0, 0],[2, 0, 1, 0, 0],[2, 0, 1, 1, 0],[3, 0, 0, 0, 0],[3, 0, 0, 1, 0]],
[[1, 0, 1, 0, 1],[2, 1, 0, 0, 0],[3, 0, 0, 0, 1],[3, 0, 0, 1, 1],[3, 1, 1, 1, 1]],
[[2, 1, 1, 0, 0],[2, 1, 1, 0, 1],[2, 1, 1, 1, 0],[3, 0, 1, 0, 0],[3, 1, 0, 1, 1]],
[[2, 1, 0, 0, 1],[2, 1, 0, 1, 0],[2, 1, 0, 1, 1],[3, 0, 1, 0, 1],[3, 1, 1, 0, 1]],
[[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[0, 1, 1, 1, 1],[1, 1, 0, 1, 1],[3, 0, 1, 1, 1]],
[[1, 1, 1, 1, 1],[2, 1, 1, 1, 1],[3, 0, 1, 1, 0],[3, 1, 0, 0, 0],[3, 1, 1, 1, 0]]
]
A partition for sets of sizes: 11*3  0*4  6*5

[4, 2, 2, 2, 2]
[8, 1, 7]
[
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 0],[0, 0, 0, 1, 1]],
[[0, 0, 1, 0, 0],[0, 1, 0, 0, 0],[0, 1, 1, 0, 0]],
[[0, 0, 1, 1, 1],[0, 1, 0, 0, 1],[0, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0],[1, 0, 0, 0, 0],[3, 1, 0, 1, 0]],
[[0, 1, 1, 0, 1],[1, 0, 0, 0, 1],[3, 1, 1, 0, 0]],
[[0, 1, 0, 1, 1],[1, 0, 0, 1, 0],[3, 1, 0, 0, 1]],
[[1, 0, 0, 1, 1],[1, 0, 1, 0, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[1, 1, 0, 0, 1],[1, 1, 0, 1, 0],[1, 1, 1, 0, 0],[1, 1, 1, 1, 1]],
[[1, 1, 1, 0, 1],[1, 1, 1, 1, 0],[2, 0, 0, 0, 0],[2, 0, 1, 0, 1],[2, 0, 1, 1, 0]],
[[2, 0, 0, 1, 0],[2, 0, 0, 1, 1],[2, 0, 1, 0, 0],[3, 0, 0, 0, 0],[3, 0, 1, 0, 1]],
[[1, 0, 1, 0, 1],[2, 1, 0, 0, 0],[3, 0, 0, 0, 1],[3, 0, 0, 1, 0],[3, 1, 1, 1, 0]],
[[2, 1, 1, 0, 0],[2, 1, 1, 0, 1],[2, 1, 1, 1, 0],[3, 0, 1, 0, 0],[3, 1, 0, 1, 1]],
[[2, 1, 0, 0, 1],[2, 1, 0, 1, 0],[2, 1, 1, 1, 1],[3, 0, 0, 1, 1],[3, 1, 1, 1, 1]],
[[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[0, 1, 1, 1, 1],[1, 1, 0, 1, 1],[3, 0, 1, 1, 1]],
[[1, 1, 0, 0, 0],[2, 1, 0, 1, 1],[3, 0, 1, 1, 0],[3, 1, 0, 0, 0],[3, 1, 1, 0, 1]]
]
A partition for sets of sizes:  8*3  1*4  7*5

[4, 2, 2, 2, 2]
[6, 0, 9]
[
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 0],[0, 0, 0, 1, 1]],
[[0, 0, 1, 0, 0],[0, 1, 0, 0, 0],[0, 1, 1, 0, 0]],
[[0, 0, 1, 1, 1],[0, 1, 0, 0, 1],[0, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0],[1, 0, 0, 0, 0],[3, 1, 0, 1, 0]],
[[0, 1, 1, 0, 1],[1, 0, 0, 0, 1],[3, 1, 1, 0, 0]],
[[0, 1, 0, 1, 1],[1, 0, 0, 1, 0],[3, 1, 0, 0, 1]],
[[1, 0, 0, 1, 1],[1, 0, 1, 0, 0],[1, 0, 1, 0, 1],[2, 0, 0, 0, 0],[3, 0, 0, 1, 0]],
[[1, 1, 0, 0, 0],[1, 1, 0, 0, 1],[1, 1, 0, 1, 0],[2, 0, 0, 1, 1],[3, 1, 0, 0, 0]],
[[1, 1, 1, 0, 1],[1, 1, 1, 1, 0],[1, 1, 1, 1, 1],[2, 0, 0, 0, 1],[3, 1, 1, 0, 1]],
[[2, 0, 0, 1, 0],[2, 0, 1, 0, 0],[2, 0, 1, 0, 1],[3, 0, 0, 0, 0],[3, 0, 0, 1, 1]],
[[2, 0, 1, 1, 1],[2, 1, 0, 0, 0],[2, 1, 0, 0, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 1]],
[[2, 1, 1, 0, 0],[2, 1, 1, 0, 1],[2, 1, 1, 1, 0],[3, 0, 1, 0, 0],[3, 1, 0, 1, 1]],
[[1, 0, 1, 1, 1],[2, 1, 0, 1, 1],[3, 0, 1, 0, 1],[3, 0, 1, 1, 0],[3, 1, 1, 1, 1]],
[[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[0, 1, 1, 1, 1],[2, 0, 1, 1, 0],[2, 1, 0, 1, 0]],
[[1, 0, 1, 1, 0],[1, 1, 0, 1, 1],[1, 1, 1, 0, 0],[2, 1, 1, 1, 1],[3, 1, 1, 1, 0]]
]
A partition for sets of sizes:  6*3  0*4  9*5
```

```
[4, 2, 2, 2, 2]
[3, 1, 10]
[
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 0],[0, 0, 0, 1, 1]],
[[0, 0, 1, 0, 0],[0, 1, 0, 0, 0],[0, 1, 1, 0, 0]],
[[0, 0, 1, 1, 1],[0, 1, 0, 0, 1],[0, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0],[0, 1, 0, 1, 1],[1, 0, 0, 0, 0],[3, 0, 0, 0, 1]],
[[0, 0, 1, 0, 1],[0, 1, 1, 1, 1],[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 1, 0, 0, 1]],
[[1, 0, 0, 1, 1],[1, 0, 1, 0, 0],[1, 0, 1, 0, 1],[2, 0, 0, 0, 0],[3, 0, 0, 1, 0]],
[[1, 1, 0, 0, 0],[1, 1, 0, 0, 1],[1, 1, 0, 1, 0],[2, 0, 0, 0, 1],[3, 1, 0, 1, 0]],
[[1, 1, 1, 0, 1],[1, 1, 1, 1, 0],[1, 1, 1, 1, 1],[2, 0, 0, 1, 0],[3, 1, 1, 1, 0]],
[[1, 0, 1, 1, 0],[2, 0, 0, 1, 1],[3, 0, 0, 0, 0],[3, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[2, 0, 1, 1, 1],[2, 1, 0, 0, 0],[2, 1, 0, 1, 0],[3, 1, 0, 0, 0],[3, 1, 1, 0, 1]],
[[2, 1, 1, 0, 0],[2, 1, 1, 0, 1],[2, 1, 1, 1, 0],[3, 0, 1, 0, 0],[3, 1, 0, 1, 1]],
[[0, 1, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 1, 0, 1],[2, 0, 1, 1, 0],[3, 1, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 1, 1, 0, 0],[2, 0, 1, 0, 0],[2, 1, 0, 1, 1],[3, 0, 1, 0, 1]],
[[1, 1, 0, 1, 1],[2, 1, 1, 1, 1],[3, 0, 1, 1, 1],[3, 1, 1, 0, 0],[3, 1, 1, 1, 1]]
]
A partition for sets of sizes:  3*3  1*4 10*5
```

```
[4, 2, 2, 2, 2]
[1, 0, 12]
[
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 0],[0, 0, 0, 1, 1]],
[[0, 0, 1, 0, 0],[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[0, 1, 0, 0, 0],[0, 1, 1, 1, 1]],
[[0, 1, 0, 0, 1],[0, 1, 0, 1, 0],[0, 1, 0, 1, 1],[1, 0, 0, 0, 0],[3, 1, 0, 0, 0]],
[[0, 1, 1, 1, 0],[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1],[1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1],[1, 0, 1, 0, 0],[1, 0, 1, 0, 1],[1, 1, 0, 0, 1],[1, 1, 1, 1, 1]],
[[1, 1, 0, 0, 0],[1, 1, 0, 1, 0],[1, 1, 0, 1, 1],[2, 0, 0, 0, 0],[3, 1, 0, 0, 1]],
[[1, 1, 1, 0, 1],[2, 0, 0, 0, 1],[3, 0, 0, 0, 0],[3, 0, 0, 0, 1],[3, 1, 1, 0, 1]],
[[2, 0, 0, 1, 0],[2, 0, 0, 1, 1],[2, 0, 1, 0, 0],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[2, 0, 1, 1, 1],[2, 1, 0, 0, 0],[2, 1, 0, 0, 1],[3, 0, 0, 1, 1],[3, 0, 1, 0, 1]],
[[2, 1, 1, 0, 0],[2, 1, 1, 0, 1],[2, 1, 1, 1, 0],[3, 0, 1, 0, 0],[3, 1, 0, 1, 1]],
[[1, 0, 1, 1, 0],[2, 0, 1, 0, 1],[3, 0, 1, 1, 0],[3, 1, 0, 1, 0],[3, 1, 1, 1, 1]],
[[2, 0, 1, 1, 0],[2, 1, 0, 1, 1],[2, 1, 1, 1, 1],[3, 1, 1, 0, 0],[3, 1, 1, 1, 0]],
[[0, 1, 1, 0, 0],[0, 1, 1, 0, 1],[1, 0, 1, 1, 1],[1, 1, 1, 0, 0],[2, 1, 0, 1, 0]]
]
A partition for sets of sizes:  1*3  0*4 12*5
```

D Zero sum partitions of $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^5\right)^*$ (with with $b \leq 15$)

```
[4, 2, 2, 2, 2, 2]
[41, 1, 0]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1, 1],[1, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 1]],
[[2, 1, 0, 0, 1, 1],[3, 0, 1, 0, 1, 0],[3, 1, 1, 0, 0, 1]],
[[2, 0, 1, 0, 0, 0],[3, 1, 0, 1, 0, 0],[3, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 1],[3, 1, 1, 0, 1, 0],[3, 1, 1, 1, 1, 1]],
[[1, 0, 0, 0, 1, 1],[1, 1, 0, 1, 0, 1],[2, 1, 0, 1, 1, 0]],
[[2, 1, 0, 1, 1, 1],[3, 0, 1, 1, 0, 0],[3, 1, 1, 0, 1, 1]],
[[0, 1, 0, 1, 0, 1],[2, 0, 1, 0, 1, 0],[2, 1, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 1],[3, 0, 1, 0, 0, 1],[3, 1, 1, 0, 0, 0]],
[[0, 0, 0, 1, 1, 0],[1, 0, 1, 1, 0, 1],[3, 0, 1, 0, 1, 1]],
[[0, 1, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[2, 1, 1, 1, 0, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 1, 0, 1, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 1, 1, 0]]
]
A partition for sets of sizes: 41*3  1*4  0*5
```

```
[4, 2, 2, 2, 2, 2]
[38, 2, 1]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1, 1],[1, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 1]],
[[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 1, 1, 0],[3, 0, 1, 0, 0, 1],[3, 1, 1, 1, 1, 1]],
[[2, 0, 0, 1, 0, 1],[3, 1, 1, 0, 0, 1],[3, 1, 1, 1, 0, 0]],
[[1, 1, 0, 1, 0, 1],[1, 1, 1, 1, 1, 1],[2, 0, 1, 0, 1, 0]],
[[2, 1, 0, 1, 1, 1],[3, 0, 1, 1, 0, 0],[3, 1, 1, 0, 1, 1]],
[[0, 1, 0, 1, 0, 1],[2, 0, 1, 0, 0, 0],[2, 1, 1, 1, 0, 1]],
[[1, 0, 0, 0, 1, 1],[2, 1, 0, 0, 1, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 0, 1, 0]],
[[0, 1, 1, 1, 0, 1],[2, 0, 1, 0, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 1, 0, 0, 0]],
[[0, 0, 0, 1, 1, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[2, 1, 0, 0, 0, 1],[3, 1, 1, 0, 1, 0]]
]
A partition for sets of sizes: 38*3  2*4  1*5
```

```
[4, 2, 2, 2, 2, 2]
[39, 0, 2]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1, 1],[1, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 1]],
[[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 1, 1, 0],[3, 0, 1, 0, 0, 1],[3, 1, 1, 1, 1, 1]],
[[2, 0, 0, 1, 0, 1],[3, 1, 1, 0, 0, 1],[3, 1, 1, 1, 0, 0]],
[[1, 1, 0, 1, 0, 1],[1, 1, 1, 1, 1, 1],[2, 0, 1, 0, 1, 0]],
[[2, 1, 0, 1, 1, 1],[3, 0, 1, 1, 0, 0],[3, 1, 1, 0, 1, 1]],
[[0, 1, 0, 1, 0, 1],[1, 0, 1, 1, 0, 1],[3, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[2, 1, 1, 1, 0, 1]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 0, 1, 1],[2, 1, 0, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 1, 1, 0]],
[[2, 0, 1, 0, 0, 0],[2, 0, 1, 0, 1, 1],[2, 1, 0, 0, 1, 1],[3, 0, 1, 0, 1, 0],[3, 1, 1, 0, 1, 0]]
]
A partition for sets of sizes: 39*3  0*4  2*5
```

```
[4, 2, 2, 2, 2, 2]
[35, 3, 2]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1, 1],[1, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 1]],
[[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 1, 1, 0],[3, 0, 1, 0, 0, 1],[3, 1, 1, 1, 1, 1]],
[[2, 0, 0, 1, 0, 1],[3, 1, 1, 0, 0, 1],[3, 1, 1, 1, 0, 0]],
[[3, 0, 1, 1, 0, 0],[3, 0, 1, 1, 1, 0],[3, 1, 1, 0, 0, 0],[3, 1, 1, 0, 1, 0]],
[[1, 1, 0, 1, 0, 1],[2, 1, 0, 0, 1, 1],[2, 1, 1, 1, 0, 1],[3, 1, 1, 0, 1, 1]],
[[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[2, 0, 1, 0, 0, 0]],
[[1, 0, 0, 0, 1, 1],[1, 1, 1, 1, 1, 1],[2, 1, 0, 0, 0, 1],[2, 1, 0, 1, 1, 1],[2, 1, 1, 0, 1, 0]],
[[0, 0, 0, 1, 1, 0],[1, 0, 1, 1, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 1, 0, 1, 0]]
]
A partition for sets of sizes: 35*3  3*4  2*5
```

```
[4, 2, 2, 2, 2, 2]
[36, 1, 3]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1, 1],[1, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 1]],
[[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 1, 1, 0],[3, 0, 1, 0, 0, 1],[3, 1, 1, 1, 1, 1]],
[[2, 0, 0, 1, 0, 1],[3, 1, 1, 0, 0, 1],[3, 1, 1, 1, 0, 0]],
[[1, 1, 0, 1, 0, 1],[1, 1, 1, 1, 1, 1],[2, 0, 1, 0, 1, 0]],
[[1, 0, 0, 0, 1, 1],[2, 0, 1, 0, 0, 0],[2, 1, 0, 0, 1, 1],[3, 1, 1, 0, 0, 0]],
[[2, 1, 0, 1, 1, 1],[2, 1, 1, 0, 1, 0],[2, 1, 1, 1, 0, 1],[3, 0, 1, 0, 1, 0],[3, 1, 1, 0, 1, 0]],
[[0, 0, 0, 1, 1, 0],[0, 1, 0, 1, 0, 1],[2, 1, 0, 0, 0, 1],[3, 0, 1, 1, 0, 0],[3, 0, 1, 1, 1, 0]],
[[0, 1, 1, 1, 0, 1],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[2, 0, 1, 0, 1, 1],[3, 1, 1, 0, 1, 1]]
]
A partition for sets of sizes: 36*3  1*4  3*5
```

```
[4, 2, 2, 2, 2, 2]
[33, 2, 4]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1, 1],[1, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 1]],
[[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 1],[3, 1, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 0],[2, 0, 1, 0, 1, 0],[2, 1, 0, 0, 0, 1],[2, 1, 0, 0, 1, 1]],
[[3, 0, 1, 0, 0, 1],[3, 0, 1, 0, 1, 0],[3, 1, 1, 0, 0, 0],[3, 1, 1, 0, 1, 1]],
[[2, 0, 0, 1, 0, 1],[2, 0, 1, 0, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 1, 1, 0, 0],[3, 1, 1, 1, 1, 1]],
[[0, 1, 0, 1, 0, 1],[1, 0, 0, 0, 1, 1],[2, 1, 0, 1, 1, 0],[2, 1, 1, 0, 1, 0],[3, 1, 1, 0, 1, 0]],
[[0, 0, 0, 1, 1, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[2, 1, 0, 1, 1, 1],[3, 1, 1, 1, 0, 0]],
[[0, 1, 1, 1, 0, 1],[1, 1, 0, 1, 0, 1],[1, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 1, 0, 0, 1]]
]
A partition for sets of sizes: 33*3  2*4  4*5
```

```
[4, 2, 2, 2, 2, 2]
[34, 0, 5]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1, 1],[1, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 1]],
[[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 1, 1, 0],[3, 0, 1, 0, 0, 1],[3, 1, 1, 1, 1, 1]],
[[2, 0, 1, 0, 0, 0],[2, 0, 1, 0, 1, 1],[2, 1, 0, 1, 1, 1],[3, 0, 1, 1, 0, 0],[3, 1, 1, 0, 0, 0]],
[[2, 0, 0, 1, 0, 1],[2, 1, 0, 0, 0, 1],[2, 1, 0, 0, 1, 1],[3, 1, 1, 0, 1, 1],[3, 1, 1, 1, 0, 0]],
[[2, 0, 0, 0, 0, 0],[2, 0, 1, 0, 1, 0],[2, 1, 1, 1, 0, 1],[3, 0, 1, 1, 1, 0],[3, 1, 1, 0, 0, 1]],
[[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 0, 1],[1, 0, 1, 1, 0, 1],[1, 1, 1, 1, 1, 1],[2, 1, 1, 0, 1, 0]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 0, 1, 1],[1, 1, 0, 1, 0, 1],[3, 0, 1, 0, 1, 0],[3, 1, 1, 0, 1, 0]]
]
A partition for sets of sizes: 34*3  0*4  5*5
```

```
[4, 2, 2, 2, 2, 2]
[30, 3, 5]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 0, 0, 1, 0, 1],[2, 0, 1, 0, 1, 0],[2, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 1]],
[[1, 0, 1, 1, 0, 1],[2, 1, 1, 0, 1, 1],[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 0, 1]],
[[2, 0, 0, 0, 0, 0],[2, 0, 1, 0, 1, 1],[2, 1, 0, 0, 0, 1],[2, 1, 1, 0, 1, 0]],
[[2, 0, 1, 0, 0, 0],[2, 1, 0, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 1, 1, 0, 0],[3, 0, 1, 1, 1, 0]],
[[1, 1, 0, 1, 0, 1],[2, 1, 0, 0, 1, 1],[3, 0, 1, 0, 1, 1],[3, 1, 0, 1, 0, 0],[3, 1, 1, 0, 0, 1]],
[[0, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 0],[3, 1, 0, 1, 1, 0]],
[[0, 0, 0, 1, 1, 0],[3, 1, 1, 0, 0, 0],[3, 1, 1, 0, 1, 0],[3, 1, 1, 0, 1, 1],[3, 1, 1, 1, 1, 1]],
[[0, 1, 1, 1, 0, 1],[1, 0, 0, 0, 1, 1],[1, 0, 1, 1, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 1, 1, 0, 0]]
]
A partition for sets of sizes: 30*3  3*4  5*5
```

```
[4, 2, 2, 2, 2, 2]
[31, 1, 6]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1, 1],[2, 0, 1, 0, 1, 0],[2, 1, 1, 1, 1, 1],[3, 1, 1, 0, 1, 0]],
[[1, 1, 0, 1, 1, 0],[2, 1, 0, 0, 1, 1],[3, 0, 1, 0, 0, 1],[3, 1, 0, 1, 0, 0],[3, 1, 1, 0, 0, 0]],
[[2, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 1, 0, 1],[3, 0, 1, 0, 1, 1],[3, 1, 1, 0, 0, 1]],
[[2, 0, 0, 1, 0, 1],[2, 1, 0, 0, 0, 1],[2, 1, 0, 1, 1, 1],[3, 1, 1, 1, 0, 0],[3, 1, 1, 1, 1, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 1, 0],[3, 0, 1, 1, 1, 0],[3, 1, 1, 0, 1, 1]],
[[0, 0, 0, 1, 1, 0],[0, 1, 0, 1, 0, 1],[1, 0, 1, 1, 0, 1],[1, 1, 0, 1, 0, 1],[2, 0, 1, 0, 1, 1]],
[[0, 1, 1, 1, 0, 1],[1, 0, 0, 0, 1, 1],[2, 0, 1, 0, 0, 0],[2, 1, 1, 0, 1, 0],[3, 0, 1, 1, 0, 0]]
]
A partition for sets of sizes: 31*3  1*4  6*5
```

```
[4, 2, 2, 2, 2, 2]
[28, 2, 7]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[2, 1, 0, 1, 1, 0],[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 1, 1]],
[[2, 0, 0, 1, 0, 1],[2, 0, 1, 0, 0, 0],[2, 1, 0, 1, 1, 1],[2, 1, 1, 0, 1, 0]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 0, 1, 1, 0],[3, 0, 1, 0, 1, 0],[3, 1, 1, 1, 1, 0]],
[[1, 1, 0, 1, 1, 0],[2, 1, 0, 0, 1, 1],[3, 0, 0, 1, 1, 1],[3, 0, 1, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[2, 0, 1, 0, 1, 1],[2, 1, 1, 0, 0, 1],[2, 1, 1, 1, 1, 1],[3, 1, 0, 0, 1, 0],[3, 1, 1, 1, 1, 1]],
[[1, 1, 0, 1, 0, 1],[2, 1, 0, 0, 0, 1],[3, 0, 1, 1, 1, 0],[3, 1, 0, 1, 1, 0],[3, 1, 1, 1, 0, 0]],
[[1, 1, 1, 1, 1, 1],[2, 0, 1, 0, 1, 0],[3, 1, 0, 1, 0, 0],[3, 1, 1, 0, 0, 0],[3, 1, 1, 0, 0, 1]],
[[0, 0, 0, 1, 1, 0],[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 0, 1],[1, 0, 0, 0, 1, 1],[3, 0, 1, 1, 0, 1]],
[[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[3, 0, 1, 1, 0, 0],[3, 1, 1, 0, 1, 0],[3, 1, 1, 0, 1, 1]]
]
A partition for sets of sizes: 28*3  2*4  7*5
```

```
[4, 2, 2, 2, 2, 2]
[29, 0, 8]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 1]],
[[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 0, 1, 1, 0],[3, 0, 1, 0, 1, 0],[3, 1, 1, 1, 1, 0]],
[[1, 1, 0, 1, 1, 0],[2, 1, 0, 0, 1, 1],[3, 0, 1, 0, 0, 1],[3, 1, 0, 1, 0, 0],[3, 1, 1, 0, 0, 0]],
[[2, 1, 0, 1, 1, 0],[2, 1, 1, 0, 1, 1],[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 1],[3, 1, 1, 0, 0, 1]],
[[1, 1, 0, 1, 0, 1],[2, 0, 1, 0, 1, 1],[3, 0, 1, 1, 0, 0],[3, 0, 1, 1, 1, 0],[3, 1, 1, 1, 0, 0]],
[[0, 1, 0, 1, 0, 1],[1, 1, 1, 1, 1, 1],[2, 0, 0, 0, 0, 0],[2, 1, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 0, 1, 1],[2, 0, 1, 0, 0, 0],[2, 1, 0, 1, 1, 1],[3, 1, 1, 0, 1, 0]],
[[0, 1, 1, 1, 0, 1],[1, 0, 1, 1, 0, 1],[2, 0, 0, 1, 0, 1],[2, 0, 1, 0, 1, 0],[3, 1, 1, 1, 1, 1]]
]
A partition for sets of sizes: 29*3  0*4  8*5
```

```
[4, 2, 2, 2, 2, 2]
[25, 3, 8]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[1, 1, 0, 1, 1, 0],[2, 0, 1, 1, 0, 1],[2, 1, 0, 0, 0, 0],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 1, 0, 1],[2, 1, 0, 0, 0, 1],[2, 1, 0, 0, 1, 0],[3, 1, 0, 1, 1, 0]],
[[1, 1, 1, 1, 1, 1],[2, 1, 0, 1, 0, 1],[2, 1, 0, 1, 1, 0],[3, 1, 1, 1, 0, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 0, 1, 0],[3, 1, 1, 0, 0, 1]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 0, 0, 1, 1],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 0],[2, 1, 1, 0, 1, 1],[2, 1, 1, 1, 1, 1],[3, 0, 0, 1, 1, 0],[3, 0, 1, 0, 1, 0]],
[[2, 0, 1, 0, 1, 1],[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 1, 1],[3, 1, 0, 0, 0, 1],[3, 1, 1, 1, 1, 0]],
[[1, 0, 0, 0, 1, 1],[2, 0, 1, 0, 1, 0],[3, 0, 1, 1, 1, 0],[3, 1, 0, 0, 1, 0],[3, 1, 0, 1, 0, 1]],
[[1, 0, 1, 1, 0, 1],[2, 0, 0, 1, 0, 1],[3, 0, 1, 1, 0, 1],[3, 1, 1, 0, 1, 0],[3, 1, 1, 1, 1, 1]],
[[0, 0, 0, 1, 1, 0],[3, 0, 1, 0, 0, 1],[3, 0, 1, 1, 0, 0],[3, 1, 1, 0, 0, 0],[3, 1, 1, 0, 1, 1]],
[[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[3, 0, 0, 1, 1, 1],[3, 0, 1, 1, 1, 1]]
]
A partition for sets of sizes: 25*3  3*4  8*5
```

```
[4, 2, 2, 2, 2, 2]
[26, 1, 9]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 0, 0, 0, 0, 0],[2, 0, 0, 1, 0, 1],[2, 1, 0, 0, 1, 0],[2, 1, 0, 1, 1, 1]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 1],[2, 1, 0, 1, 1, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 0, 0, 0]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 0, 1, 1, 0],[3, 0, 1, 0, 1, 0],[3, 1, 1, 1, 1, 0]],
[[1, 1, 0, 1, 1, 0],[2, 1, 1, 0, 1, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[2, 0, 1, 0, 0, 0],[2, 0, 1, 0, 1, 1],[2, 1, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1],[3, 1, 1, 0, 0, 1]],
[[1, 1, 0, 1, 0, 1],[2, 1, 0, 0, 0, 0],[3, 0, 1, 1, 1, 0],[3, 1, 0, 1, 0, 0],[3, 1, 1, 1, 1, 1]],
[[1, 1, 1, 1, 1, 1],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 0, 0],[3, 1, 0, 1, 1, 0],[3, 1, 1, 0, 1, 0]],
[[0, 1, 0, 1, 0, 1],[1, 0, 0, 0, 1, 1],[1, 0, 1, 1, 0, 1],[3, 0, 1, 0, 0, 1],[3, 1, 0, 0, 1, 0]],
[[0, 0, 0, 1, 1, 0],[0, 1, 1, 1, 0, 1],[2, 0, 1, 0, 1, 0],[3, 0, 1, 1, 0, 1],[3, 1, 1, 1, 0, 0]]
]
A partition for sets of sizes: 26*3  1*4  9*5
```

```
[4, 2, 2, 2, 2, 2]
[23, 2, 10]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[2, 0, 1, 0, 0, 0],[2, 1, 0, 0, 0, 1],[2, 1, 1, 1, 0, 1]],
[[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[2, 1, 0, 0, 1, 0],[2, 1, 0, 0, 1, 1]],
[[1, 1, 0, 1, 0, 1],[2, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 0, 1, 0, 0]],
[[2, 0, 1, 1, 0, 1],[2, 1, 0, 1, 0, 1],[2, 1, 0, 1, 1, 0],[3, 0, 0, 0, 1, 0],[3, 0, 1, 1, 0, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 0, 0, 0]],
[[2, 0, 1, 0, 0, 1],[2, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 0, 0],[3, 0, 1, 1, 1, 0]],
[[1, 1, 1, 1, 1, 1],[2, 1, 1, 0, 1, 1],[3, 0, 0, 1, 0, 1],[3, 0, 0, 1, 1, 0],[3, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 1],[2, 1, 0, 1, 1, 1],[3, 0, 1, 0, 0, 1],[3, 0, 1, 0, 1, 0],[3, 1, 1, 0, 1, 1]],
[[1, 0, 1, 1, 0, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 1, 0, 0],[3, 1, 0, 0, 1, 0],[3, 1, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 0, 0, 1],[3, 1, 0, 1, 1, 0],[3, 1, 1, 1, 1, 1]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 0, 1, 1],[1, 1, 0, 1, 1, 0],[3, 0, 1, 1, 1, 1],[3, 1, 1, 1, 0, 0]],
[[0, 1, 1, 1, 0, 1],[3, 0, 1, 0, 1, 1],[3, 1, 0, 1, 0, 1],[3, 1, 1, 0, 0, 1],[3, 1, 1, 0, 1, 0]]
]
A partition for sets of sizes: 23*3  2*4 10*5
```

```
[4, 2, 2, 2, 2, 2]
[24, 0, 11]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[1, 1, 0, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 1, 1, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 1]],
[[1, 1, 0, 1, 0, 1],[2, 0, 0, 0, 0, 0],[3, 0, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 1],[2, 1, 0, 1, 1, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 1, 1],[3, 1, 1, 1, 0, 0]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 0, 1, 1, 0],[3, 0, 1, 0, 1, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 0, 0, 0, 0],[2, 1, 0, 0, 0, 1],[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 0, 0],[3, 1, 0, 1, 1, 0]],
[[1, 1, 1, 1, 1, 1],[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 0],[3, 1, 0, 0, 0, 1],[3, 1, 1, 0, 0, 1]],
[[2, 0, 1, 1, 0, 1],[2, 1, 0, 0, 1, 0],[2, 1, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0],[3, 1, 1, 0, 1, 0]],
[[1, 0, 1, 1, 0, 1],[2, 0, 0, 1, 0, 1],[3, 0, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 0, 1, 1, 1, 0]],
[[0, 0, 0, 1, 1, 0],[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 0, 1],[1, 1, 0, 1, 1, 0],[3, 1, 1, 0, 0, 0]],
[[1, 0, 0, 0, 1, 1],[2, 0, 1, 0, 0, 0],[3, 0, 1, 1, 1, 1],[3, 1, 1, 0, 1, 1],[3, 1, 1, 1, 1, 1]]
]
A partition for sets of sizes: 24*3  0*4 11*5
```

```
[4, 2, 2, 2, 2, 2]
[20, 3, 11]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 1, 1, 1, 0, 1],[1, 1, 1, 1, 1, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 1, 0],[1, 1, 0, 0, 1, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 0, 1]],
[[2, 0, 0, 1, 0, 1],[2, 0, 1, 0, 0, 0],[2, 1, 0, 0, 0, 0],[2, 1, 1, 1, 0, 1]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 0, 1, 0]],
[[1, 1, 0, 1, 0, 1],[2, 0, 1, 1, 1, 1],[3, 0, 0, 1, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 1, 0, 1, 1]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 0],[3, 1, 0, 1, 0, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 1, 1],[3, 1, 1, 1, 0, 0]],
[[1, 1, 1, 0, 1, 1],[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 0, 0],[3, 1, 0, 0, 1, 0],[3, 1, 1, 1, 1, 0]],
[[1, 1, 0, 1, 1, 0],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 0, 1],[3, 0, 1, 1, 1, 0],[3, 1, 0, 0, 0, 1]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 0],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 0, 1]],
[[1, 0, 1, 1, 0, 1],[2, 1, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1],[3, 0, 1, 1, 1, 1],[3, 1, 1, 0, 0, 0]],
[[0, 1, 0, 1, 0, 1],[2, 0, 1, 1, 0, 1],[2, 1, 0, 0, 1, 0],[2, 1, 0, 1, 1, 0],[2, 1, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 0, 1, 1],[2, 0, 0, 1, 0, 0],[2, 1, 0, 1, 1, 1],[3, 1, 0, 1, 1, 0]],
[[0, 1, 1, 1, 0, 1],[1, 0, 0, 1, 1, 1],[1, 1, 1, 1, 1, 1],[3, 1, 1, 0, 1, 0],[3, 1, 1, 1, 1, 1]]
]
A partition for sets of sizes: 20*3  3*4 11*5
```

```
[4, 2, 2, 2, 2, 2]
[21, 1, 12]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 1, 0, 1, 0, 0]],
[[2, 0, 0, 0, 0, 0],[2, 0, 0, 1, 0, 0],[2, 0, 1, 0, 0, 1],[2, 0, 1, 1, 0, 1]],
[[1, 1, 1, 1, 0, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 0, 1]],
[[1, 1, 0, 0, 1, 0],[2, 0, 1, 0, 1, 0],[3, 0, 0, 0, 1, 0],[3, 0, 0, 1, 0, 0],[3, 1, 1, 1, 1, 0]],
[[1, 1, 0, 1, 0, 1],[2, 0, 1, 1, 1, 1],[3, 0, 0, 0, 1, 1],[3, 0, 0, 1, 0, 1],[3, 1, 1, 1, 0, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 1],[2, 1, 0, 1, 1, 0],[3, 0, 0, 1, 1, 0],[3, 1, 0, 1, 1, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 0, 0, 1],[3, 1, 0, 0, 1, 0]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 0, 1, 0, 1, 0],[3, 1, 1, 1, 1, 1]],
[[2, 0, 1, 0, 1, 1],[2, 1, 0, 0, 0, 0],[2, 1, 0, 0, 0, 1],[3, 1, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[2, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 0, 0],[3, 1, 0, 1, 0, 0]],
[[1, 1, 1, 0, 1, 1],[2, 1, 0, 1, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 1, 0, 0, 0],[3, 1, 1, 0, 1, 0]],
[[0, 1, 0, 1, 0, 1],[1, 0, 0, 1, 1, 1],[2, 0, 1, 0, 0, 0],[2, 1, 0, 0, 1, 0],[3, 0, 1, 0, 0, 0]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 0, 1, 1],[1, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 1],[3, 1, 0, 1, 0, 1]],
[[0, 1, 1, 1, 0, 1],[1, 0, 1, 1, 0, 1],[1, 1, 0, 1, 1, 0],[3, 0, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1]]
]
A partition for sets of sizes: 21*3  1*4 12*5
```

```
[4, 2, 2, 2, 2, 2]
[18, 2, 13]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[3, 0, 0, 0, 0, 1],[3, 0, 0, 1, 1, 0]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 0, 0, 0],[3, 0, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 1, 1, 0]],
[[2, 0, 0, 1, 0, 0],[2, 0, 0, 1, 0, 1],[2, 0, 0, 1, 1, 0],[3, 0, 1, 0, 0, 0],[3, 0, 1, 1, 1, 1]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 1, 0, 0],[3, 0, 1, 1, 0, 0]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 0, 0, 1],[3, 1, 0, 0, 1, 0]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 1, 0, 0, 0, 1],[3, 1, 1, 0, 0, 0],[3, 1, 1, 0, 1, 1]],
[[1, 0, 1, 0, 1, 0],[1, 1, 0, 0, 1, 0],[1, 1, 0, 1, 1, 0],[2, 1, 0, 0, 0, 1],[3, 1, 1, 1, 1, 1]],
[[2, 1, 0, 1, 1, 0],[2, 1, 1, 0, 1, 1],[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 1],[3, 1, 1, 0, 0, 1]],
[[2, 0, 1, 1, 0, 1],[2, 1, 0, 1, 1, 1],[2, 1, 1, 1, 0, 0],[3, 1, 1, 0, 1, 0],[3, 1, 1, 1, 0, 0]],
[[1, 0, 1, 1, 0, 1],[1, 1, 0, 1, 0, 1],[1, 1, 1, 0, 1, 0],[2, 0, 1, 0, 0, 0],[3, 0, 1, 0, 1, 0]],
[[0, 0, 0, 1, 1, 0],[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 0, 1],[1, 0, 0, 0, 1, 1],[3, 0, 1, 1, 0, 1]],
[[1, 0, 0, 1, 1, 1],[1, 1, 1, 1, 0, 1],[1, 1, 1, 1, 1, 0],[2, 1, 0, 0, 1, 0],[3, 1, 0, 1, 1, 0]]
]
A partition for sets of sizes: 18*3  2*4 13*5
```

```
[4, 2, 2, 2, 2, 2]
[19, 0, 14]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 1],[2, 0, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 0, 1, 1, 0, 0]],
[[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 0, 0],[3, 1, 1, 0, 0, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 1, 1, 0]],
[[1, 1, 1, 1, 0, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 0, 1, 0, 1],[3, 1, 1, 1, 1, 1]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 0, 1, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 1, 0, 1],[2, 0, 1, 1, 1, 1],[3, 0, 0, 1, 0, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 0, 0, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 0, 0, 1],[3, 1, 0, 0, 1, 0]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 1, 0, 0, 0],[3, 0, 1, 1, 0, 0],[3, 1, 0, 1, 1, 0]],
[[2, 0, 0, 1, 1, 0],[2, 1, 0, 0, 0, 0],[2, 1, 0, 0, 0, 1],[3, 0, 1, 0, 1, 0],[3, 0, 1, 1, 0, 1]],
[[2, 1, 0, 1, 1, 0],[2, 1, 1, 0, 1, 1],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 1, 1, 0, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 0, 1, 1, 0],[1, 1, 1, 1, 1, 0],[2, 0, 1, 1, 0, 1],[3, 0, 1, 1, 1, 1]],
[[0, 1, 0, 1, 0, 1],[1, 0, 1, 1, 0, 1],[1, 1, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 0, 1, 1],[2, 0, 1, 0, 0, 0],[2, 1, 0, 1, 1, 1],[3, 1, 1, 0, 1, 0]],
[[0, 1, 1, 1, 0, 1],[1, 0, 0, 1, 1, 1],[2, 1, 0, 0, 1, 0],[2, 1, 1, 1, 0, 0],[3, 1, 0, 1, 0, 0]]
]
A partition for sets of sizes: 19*3  0*4 14*5
```

```
[4, 2, 2, 2, 2, 2]
[15, 3, 14]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 0, 1, 1, 1, 1],[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1]],
[[1, 1, 0, 0, 1, 0],[1, 1, 0, 0, 1, 1],[1, 1, 0, 1, 0, 0],[1, 1, 0, 1, 0, 1]],
[[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 0],[1, 1, 1, 0, 0, 1]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 0, 1, 0],[3, 1, 1, 1, 1, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 1, 1, 0]],
[[2, 0, 0, 1, 0, 0],[2, 0, 0, 1, 0, 1],[2, 0, 0, 1, 1, 0],[3, 0, 0, 0, 1, 0],[3, 0, 0, 1, 0, 1]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 0, 1, 1],[3, 0, 1, 0, 1, 1]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 0],[3, 1, 0, 1, 0, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 1, 1],[3, 1, 1, 1, 0, 0]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 0],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 1, 0, 1],[2, 0, 1, 1, 0, 0],[3, 0, 1, 0, 0, 0],[3, 1, 0, 0, 0, 1],[3, 1, 1, 0, 0, 0]],
[[2, 1, 0, 0, 0, 1],[2, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0],[3, 0, 1, 1, 0, 0],[3, 1, 1, 0, 1, 0]],
[[1, 0, 1, 0, 1, 0],[2, 0, 1, 0, 0, 0],[3, 0, 1, 0, 0, 1],[3, 1, 0, 0, 1, 0],[3, 1, 1, 0, 0, 1]],
[[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 0, 1],[1, 0, 0, 0, 1, 1],[1, 1, 1, 1, 0, 1],[2, 1, 0, 1, 1, 0]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[2, 0, 1, 1, 0, 1],[2, 1, 0, 1, 1, 1],[3, 1, 1, 0, 1, 1]],
[[1, 1, 1, 1, 1, 0],[2, 1, 0, 0, 1, 0],[3, 0, 1, 1, 0, 1],[3, 0, 1, 1, 1, 0],[3, 0, 1, 1, 1, 1]]
]
A partition for sets of sizes: 15*3  3*4 14*5
```

```
[4, 2, 2, 2, 2, 2]
[16, 1, 15]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[2, 0, 0, 1, 1, 1]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 0, 0],[2, 1, 1, 1, 1, 0]],
[[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[1, 1, 0, 1, 0, 0],[1, 1, 0, 1, 1, 1]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 1, 0, 0, 0],[2, 0, 0, 0, 0, 0],[3, 1, 1, 0, 1, 1]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 0, 1, 0],[3, 1, 1, 1, 1, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 1, 0]],
[[1, 1, 0, 0, 1, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 0, 0, 1]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 0, 1, 1],[3, 0, 1, 0, 1, 1]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 0],[3, 1, 0, 1, 0, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 1, 1],[3, 1, 1, 1, 0, 0]],
[[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 1],[3, 0, 1, 0, 0, 0],[3, 0, 1, 1, 0, 0],[3, 1, 0, 1, 1, 0]],
[[2, 0, 0, 1, 1, 0],[2, 0, 1, 1, 0, 0],[2, 1, 0, 0, 0, 1],[3, 0, 1, 0, 0, 1],[3, 1, 0, 0, 1, 0]],
[[1, 0, 1, 1, 0, 1],[2, 1, 0, 1, 1, 0],[3, 0, 1, 0, 1, 0],[3, 0, 1, 1, 1, 1],[3, 1, 1, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 0, 1],[2, 0, 1, 0, 0, 0],[2, 0, 1, 1, 0, 1],[2, 1, 0, 0, 1, 0]],
[[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 1, 0, 0, 1]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[2, 1, 0, 1, 1, 1],[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1]],
[[1, 0, 0, 0, 1, 1],[1, 1, 1, 0, 0, 1],[1, 1, 1, 1, 1, 0],[2, 1, 1, 1, 0, 0],[3, 1, 1, 0, 0, 0]]
]
A partition for sets of sizes: 16*3  1*4 15*5
```

```
[4, 2, 2, 2, 2, 2]
[13, 2, 16]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[1, 0, 1, 0, 1, 0],[1, 0, 1, 0, 1, 1]],
[[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[1, 0, 1, 1, 1, 0],[1, 0, 1, 1, 1, 1]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 0, 1],[3, 1, 0, 0, 1, 0]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[2, 0, 0, 0, 0, 0],[3, 1, 0, 1, 0, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 0, 1, 0],[3, 1, 1, 1, 1, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 1, 0]],
[[1, 1, 1, 0, 0, 0],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 1, 0, 0],[3, 0, 1, 1, 0, 0]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 1, 1],[3, 1, 0, 1, 1, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 1, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 0, 1],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 0, 1],[3, 1, 0, 0, 0, 1]],
[[2, 0, 0, 1, 1, 0],[2, 0, 0, 1, 1, 1],[2, 0, 1, 1, 0, 0],[3, 0, 0, 1, 1, 0],[3, 0, 1, 0, 1, 1]],
[[2, 1, 0, 0, 0, 1],[2, 1, 0, 1, 1, 0],[2, 1, 0, 1, 1, 1],[3, 0, 1, 0, 0, 0],[3, 1, 1, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[2, 1, 0, 0, 1, 0],[2, 1, 1, 0, 1, 1],[3, 0, 1, 0, 1, 0],[3, 0, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0, 1],[1, 1, 0, 1, 0, 0],[2, 0, 1, 0, 0, 0],[2, 1, 1, 1, 0, 0],[3, 1, 0, 1, 0, 1]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 0, 1, 1],[1, 1, 0, 0, 1, 1],[3, 0, 1, 1, 1, 1],[3, 1, 1, 0, 0, 1]],
[[0, 1, 1, 1, 0, 1],[1, 0, 0, 1, 1, 1],[1, 1, 1, 0, 0, 1],[1, 1, 1, 1, 0, 1],[1, 1, 1, 1, 1, 0]]
]
A partition for sets of sizes: 13*3  2*4 16*5
```

```
[4, 2, 2, 2, 2, 2]
[14, 0, 17]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 0, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[2, 0, 0, 0, 1, 1]],
[[1, 0, 1, 0, 0, 0],[1, 0, 1, 0, 0, 1],[2, 0, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 1, 0]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[2, 0, 0, 1, 1, 0],[3, 1, 0, 0, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 0, 0],[3, 1, 1, 0, 0, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[1, 1, 1, 1, 0, 1],[2, 0, 0, 1, 1, 1],[3, 0, 0, 0, 1, 0],[3, 0, 0, 1, 0, 0],[3, 1, 1, 1, 0, 0]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 0, 1, 1],[3, 0, 1, 0, 1, 1]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 1, 0]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 0, 1, 1, 0],[3, 1, 1, 0, 1, 0]],
[[1, 1, 1, 0, 0, 0],[2, 1, 0, 0, 0, 1],[3, 0, 1, 0, 0, 0],[3, 0, 1, 1, 1, 0],[3, 0, 1, 1, 1, 1]],
[[2, 1, 0, 1, 1, 0],[2, 1, 1, 0, 1, 1],[2, 1, 1, 1, 0, 0],[3, 0, 1, 0, 0, 1],[3, 1, 1, 0, 0, 0]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 1, 0, 0],[1, 1, 1, 1, 1, 0],[2, 0, 1, 0, 0, 0],[3, 0, 1, 1, 0, 0]],
[[1, 0, 0, 0, 1, 1],[1, 1, 0, 0, 1, 1],[1, 1, 1, 0, 0, 1],[2, 1, 0, 1, 1, 1],[3, 1, 1, 1, 1, 0]],
[[0, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[2, 0, 1, 1, 0, 0],[2, 1, 0, 0, 1, 0],[3, 1, 1, 1, 1, 1]],
[[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 0, 1],[1, 0, 1, 0, 1, 0],[1, 0, 1, 1, 1, 1],[2, 0, 1, 1, 0, 1]]
]
A partition for sets of sizes: 14*3  0*4 17*5
```

```
[4, 2, 2, 2, 2, 2]
[10, 3, 17]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[1, 0, 0, 0, 1, 1],[2, 1, 1, 1, 1, 0]],
[[0, 1, 1, 1, 0, 1],[1, 0, 0, 1, 0, 0],[1, 0, 0, 1, 0, 1],[2, 1, 1, 1, 0, 0]],
[[1, 0, 0, 1, 1, 1],[1, 0, 1, 0, 0, 0],[1, 1, 0, 0, 0, 0],[1, 1, 1, 1, 1, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 1, 0]],
[[1, 0, 1, 0, 0, 1],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[2, 0, 0, 0, 1, 0],[3, 1, 0, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 0, 1, 1],[3, 1, 1, 1, 1, 0]],
[[1, 0, 1, 0, 1, 0],[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 1, 1],[2, 0, 0, 1, 0, 1],[3, 1, 0, 0, 1, 0]],
[[2, 0, 0, 1, 0, 0],[2, 0, 0, 1, 1, 0],[2, 0, 0, 1, 1, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 0, 0, 1],[3, 0, 1, 0, 0, 1]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 0],[3, 1, 0, 1, 0, 1]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 0, 1, 0],[3, 1, 0, 0, 0, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 0, 0, 0]],
[[0, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0],[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 0]],
[[2, 1, 0, 1, 1, 0],[2, 1, 0, 1, 1, 1],[2, 1, 1, 0, 1, 1],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 0, 0]],
[[2, 0, 1, 1, 0, 1],[2, 1, 0, 0, 0, 1],[2, 1, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1],[3, 1, 1, 1, 1, 1]],
[[1, 0, 1, 1, 1, 1],[2, 0, 1, 1, 0, 0],[3, 0, 1, 1, 0, 1],[3, 1, 0, 1, 0, 0],[3, 1, 1, 0, 1, 0]],
[[1, 1, 1, 1, 0, 1],[2, 0, 1, 0, 0, 0],[3, 0, 0, 1, 1, 1],[3, 0, 1, 1, 1, 1],[3, 1, 1, 1, 0, 1]],
[[0, 0, 0, 1, 1, 0],[3, 0, 1, 0, 0, 0],[3, 0, 1, 1, 0, 0],[3, 1, 1, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[0, 1, 0, 1, 0, 1],[1, 0, 0, 1, 1, 0],[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 1],[1, 1, 1, 1, 1, 0]]
]
A partition for sets of sizes: 10*3  3*4 17*5
```

```
[4, 2, 2, 2, 2, 2]
[11, 1, 18]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
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[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
[[0, 0, 1, 1, 1, 1],[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[1, 0, 0, 0, 1, 1],[1, 0, 0, 1, 0, 0],[2, 1, 0, 0, 1, 1]],
[[1, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[1, 0, 1, 0, 0, 0],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[2, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 0, 1, 1],[3, 1, 1, 1, 1, 0]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[1, 1, 1, 0, 0, 0],[2, 0, 0, 1, 1, 0],[3, 0, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 1, 1, 1]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 1, 0, 0],[3, 0, 1, 1, 0, 0]],
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[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 1, 1],[3, 1, 1, 1, 0, 0]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 0, 1, 0],[3, 1, 0, 1, 1, 0]],
[[1, 0, 1, 0, 0, 1],[2, 0, 0, 1, 1, 1],[3, 0, 1, 1, 1, 1],[3, 1, 1, 0, 0, 0],[3, 1, 1, 0, 0, 1]],
[[1, 0, 1, 0, 1, 0],[1, 0, 1, 1, 1, 0],[2, 0, 1, 1, 0, 1],[2, 1, 0, 0, 1, 0],[2, 1, 1, 0, 1, 1]],
[[1, 0, 1, 1, 1, 1],[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 0, 0],[3, 0, 1, 1, 1, 0]],
[[2, 1, 0, 1, 1, 0],[2, 1, 0, 1, 1, 1],[2, 1, 1, 1, 0, 0],[3, 0, 1, 0, 0, 0],[3, 1, 0, 1, 0, 1]],
[[0, 0, 0, 1, 1, 0],[0, 1, 1, 1, 0, 1],[1, 1, 0, 0, 1, 1],[1, 1, 1, 1, 0, 1],[2, 1, 0, 1, 0, 1]],
[[0, 1, 0, 1, 0, 1],[1, 1, 1, 1, 1, 0],[2, 0, 1, 0, 0, 0],[2, 1, 0, 0, 0, 1],[3, 1, 0, 0, 1, 0]]
]
A partition for sets of sizes: 11*3  1*4 18*5
```

```
[4, 2, 2, 2, 2, 2]
[8, 2, 19]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 0, 0, 1, 1, 0],[0, 1, 1, 1, 0, 1],[0, 1, 1, 1, 1, 0]],
[[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 0, 0, 1, 1, 0]],
[[0, 1, 0, 1, 1, 1],[1, 0, 0, 0, 1, 1],[1, 0, 0, 1, 0, 0],[1, 0, 0, 1, 0, 1],[1, 1, 0, 1, 0, 1]],
[[1, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[1, 0, 1, 0, 0, 0],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1]],
[[0, 0, 1, 1, 1, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[3, 0, 0, 0, 0, 0],[3, 0, 1, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 0, 1, 1],[3, 1, 1, 1, 1, 0]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 0],[3, 0, 0, 0, 0, 1],[3, 0, 0, 0, 1, 0],[3, 1, 1, 0, 0, 0]],
[[1, 0, 1, 0, 0, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 1, 1],[3, 0, 0, 1, 0, 1],[3, 0, 1, 0, 1, 0]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 1, 0, 0],[3, 0, 1, 1, 0, 0]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 1, 1],[3, 1, 0, 1, 1, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 1, 0, 0, 0],[3, 1, 1, 0, 1, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 1, 1, 1],[3, 1, 0, 1, 0, 0]],
[[1, 1, 1, 0, 0, 0],[2, 1, 1, 1, 1, 0],[3, 0, 1, 1, 0, 1],[3, 1, 0, 0, 0, 0],[3, 1, 1, 0, 1, 1]],
[[1, 1, 0, 0, 1, 1],[2, 0, 0, 1, 1, 1],[3, 1, 0, 0, 1, 0],[3, 1, 1, 0, 0, 1],[3, 1, 1, 1, 1, 1]],
[[1, 0, 1, 0, 1, 0],[1, 0, 1, 1, 1, 1],[2, 0, 1, 0, 0, 0],[2, 1, 0, 0, 0, 1],[2, 1, 1, 1, 0, 0]],
[[2, 0, 1, 1, 0, 1],[2, 1, 1, 0, 1, 1],[2, 1, 1, 1, 1, 1],[3, 1, 0, 1, 0, 1],[3, 1, 1, 1, 0, 0]],
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[[0, 1, 1, 0, 0, 0],[1, 1, 1, 1, 1, 0],[2, 0, 0, 1, 1, 0],[2, 1, 1, 1, 0, 1],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 0],[0, 1, 0, 1, 0, 1],[1, 0, 1, 1, 1, 0],[1, 1, 1, 1, 0, 1],[2, 1, 0, 0, 1, 0]]
]
A partition for sets of sizes:  8*3  2*4 19*5
```

```
[4, 2, 2, 2, 2, 2]
[9, 0, 20]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 1, 0]],
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[[0, 1, 0, 1, 1, 1],[1, 0, 0, 0, 1, 1],[1, 0, 0, 1, 0, 0],[1, 0, 0, 1, 0, 1],[1, 1, 0, 1, 0, 1]],
[[1, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[1, 0, 1, 0, 0, 0],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1]],
[[0, 0, 0, 1, 1, 0],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[1, 1, 1, 0, 0, 0],[1, 1, 1, 1, 1, 1]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 0, 1, 1],[3, 1, 1, 1, 1, 0]],
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[[2, 0, 0, 1, 0, 0],[2, 0, 1, 0, 0, 0],[2, 0, 1, 0, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1]],
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[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 1, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 0, 0],[3, 1, 0, 1, 1, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 0, 1]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 0, 1, 1, 1],[3, 1, 1, 0, 1, 1]],
[[2, 0, 1, 1, 0, 0],[2, 1, 0, 0, 0, 1],[2, 1, 0, 1, 1, 0],[3, 1, 0, 1, 0, 0],[3, 1, 1, 1, 1, 1]],
[[1, 0, 1, 0, 0, 1],[1, 0, 1, 0, 1, 0],[1, 0, 1, 1, 1, 1],[2, 1, 1, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[0, 1, 0, 1, 0, 0],[3, 0, 1, 0, 0, 0],[3, 0, 1, 0, 1, 0],[3, 0, 1, 1, 1, 0],[3, 1, 1, 0, 0, 0]],
[[0, 1, 0, 1, 0, 1],[1, 1, 0, 1, 0, 0],[2, 1, 0, 1, 1, 1],[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1]],
[[0, 1, 1, 0, 0, 0],[1, 1, 1, 1, 1, 0],[2, 0, 1, 1, 0, 1],[2, 1, 0, 0, 1, 0],[3, 1, 1, 0, 0, 1]],
[[1, 0, 1, 1, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 1, 1, 0, 0],[3, 1, 0, 1, 0, 1],[3, 1, 1, 1, 0, 0]]
]
A partition for sets of sizes:  9*3  0*4 20*5
```

```
[4, 2, 2, 2, 2, 2]
[5, 3, 20]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[1, 0, 0, 0, 0, 0],[3, 1, 1, 0, 0, 1]],
[[0, 1, 0, 1, 0, 0],[0, 1, 0, 1, 0, 1],[0, 1, 0, 1, 1, 0],[0, 1, 0, 1, 1, 1]],
[[0, 1, 1, 0, 0, 0],[0, 1, 1, 0, 0, 1],[0, 1, 1, 1, 1, 0],[0, 1, 1, 1, 1, 1]],
[[0, 0, 1, 1, 1, 1],[0, 1, 1, 1, 0, 1],[1, 0, 0, 0, 0, 1],[1, 0, 0, 0, 1, 0],[2, 1, 0, 0, 0, 1]],
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[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 0, 1, 1],[3, 1, 0, 0, 1, 0]],
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[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 1],[3, 0, 0, 1, 0, 1],[3, 1, 1, 1, 1, 1]],
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[[0, 0, 0, 1, 0, 1],[3, 0, 1, 1, 0, 0],[3, 0, 1, 1, 1, 1],[3, 1, 0, 0, 0, 0],[3, 1, 0, 1, 1, 0]],
[[0, 1, 0, 0, 1, 1],[1, 1, 1, 0, 0, 0],[1, 1, 1, 0, 0, 1],[3, 0, 1, 1, 1, 0],[3, 1, 1, 1, 0, 0]],
[[1, 1, 0, 0, 1, 1],[2, 1, 1, 1, 0, 0],[3, 0, 1, 0, 0, 0],[3, 1, 0, 0, 1, 1],[3, 1, 0, 1, 0, 0]]
]
A partition for sets of sizes:  5*3  3*4 20*5
```

```
[4, 2, 2, 2, 2, 2]
[6, 1, 21]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 0]],
[[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 1]],
[[0, 1, 0, 0, 1, 1],[0, 1, 0, 1, 0, 0],[0, 1, 1, 0, 0, 0],[0, 1, 1, 1, 1, 1]],
[[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 1, 1],[0, 1, 1, 1, 0, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 0, 1]],
[[0, 0, 1, 1, 1, 1],[0, 1, 1, 1, 1, 0],[1, 0, 0, 0, 0, 1],[1, 0, 0, 0, 1, 0],[2, 1, 0, 0, 1, 0]],
[[0, 0, 0, 1, 0, 1],[0, 1, 0, 1, 1, 0],[1, 0, 0, 0, 1, 1],[1, 0, 0, 1, 0, 0],[2, 1, 0, 1, 0, 0]],
[[1, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[1, 0, 1, 0, 0, 0],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 1, 1],[3, 1, 0, 0, 0, 0]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[2, 0, 0, 0, 1, 0],[3, 1, 0, 1, 1, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 0, 0],[3, 1, 1, 0, 0, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[1, 1, 1, 1, 0, 1],[2, 0, 0, 1, 1, 0],[3, 0, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 0, 1, 0]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 1, 0, 0],[3, 0, 1, 1, 0, 0]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 1],[2, 1, 0, 1, 1, 0],[3, 0, 0, 1, 1, 1],[3, 1, 0, 1, 1, 1]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 0, 1]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 0, 0, 1, 0]],
[[1, 0, 1, 0, 0, 1],[2, 0, 0, 1, 1, 1],[3, 0, 1, 0, 0, 0],[3, 1, 0, 0, 1, 1],[3, 1, 0, 1, 0, 1]],
[[1, 0, 0, 1, 0, 1],[2, 0, 1, 1, 0, 0],[3, 0, 1, 0, 1, 0],[3, 1, 1, 1, 0, 0],[3, 1, 1, 1, 1, 1]],
[[2, 0, 1, 1, 0, 1],[2, 1, 0, 1, 1, 1],[2, 1, 1, 1, 0, 0],[3, 1, 1, 0, 0, 0],[3, 1, 1, 1, 1, 0]],
[[0, 1, 0, 1, 0, 1],[1, 0, 1, 0, 1, 0],[1, 1, 0, 0, 1, 1],[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0]],
[[2, 0, 1, 0, 0, 0],[2, 1, 0, 0, 0, 1],[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 1],[3, 0, 1, 1, 1, 1]],
[[0, 0, 0, 1, 1, 0],[1, 0, 1, 1, 1, 0],[1, 0, 1, 1, 1, 1],[1, 1, 1, 0, 0, 1],[1, 1, 1, 1, 1, 0]]
]
A partition for sets of sizes:  6*3  1*4 21*5
```

```
[4, 2, 2, 2, 2, 2]
[3, 2, 22]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 0, 1, 0, 1, 1],[0, 1, 0, 0, 0, 0],[0, 1, 0, 0, 0, 1]],
[[0, 0, 0, 1, 0, 1],[0, 0, 1, 1, 1, 1],[0, 1, 0, 0, 1, 0],[0, 1, 1, 0, 0, 0]],
[[0, 0, 0, 1, 1, 0],[0, 1, 0, 0, 1, 1],[0, 1, 0, 1, 0, 0],[0, 1, 0, 1, 1, 0],[0, 1, 0, 1, 1, 1]],
[[0, 1, 0, 1, 0, 1],[0, 1, 1, 0, 0, 1],[0, 1, 1, 0, 1, 0],[1, 0, 0, 0, 0, 0],[3, 1, 0, 1, 1, 0]],
[[0, 1, 1, 1, 0, 0],[0, 1, 1, 1, 0, 1],[0, 1, 1, 1, 1, 0],[1, 0, 0, 0, 0, 1],[3, 1, 1, 1, 1, 0]],
[[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[1, 0, 0, 0, 1, 1],[1, 0, 0, 1, 0, 0],[1, 1, 1, 0, 1, 0]],
[[1, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[1, 0, 1, 0, 0, 0],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[2, 0, 0, 0, 1, 1],[3, 1, 0, 1, 1, 1]],
[[1, 0, 0, 1, 0, 1],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 0, 0],[3, 0, 0, 1, 1, 0]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[1, 1, 1, 1, 0, 1],[2, 0, 0, 1, 1, 0],[3, 0, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 0, 1, 0]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 1, 0, 0],[3, 0, 1, 1, 0, 0]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 1, 1],[3, 1, 0, 1, 0, 1]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 1, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 0, 0, 1, 0]],
[[1, 0, 1, 0, 0, 1],[2, 0, 0, 1, 1, 1],[3, 0, 1, 0, 1, 0],[3, 1, 1, 0, 0, 0],[3, 1, 1, 1, 0, 0]],
[[2, 1, 0, 1, 1, 0],[2, 1, 0, 1, 1, 1],[2, 1, 1, 1, 0, 0],[3, 0, 1, 1, 0, 1],[3, 1, 0, 0, 0, 0]],
[[2, 0, 1, 1, 0, 1],[2, 1, 0, 0, 1, 0],[2, 1, 1, 0, 1, 1],[3, 1, 1, 0, 0, 1],[3, 1, 1, 1, 0, 1]],
[[1, 0, 1, 0, 1, 0],[1, 1, 0, 0, 1, 1],[1, 1, 1, 1, 1, 0],[2, 0, 1, 0, 0, 0],[3, 0, 1, 1, 1, 1]],
[[0, 1, 1, 0, 1, 1],[1, 1, 1, 0, 0, 1],[2, 0, 1, 1, 0, 0],[2, 1, 0, 0, 0, 1],[3, 1, 1, 1, 1, 1]],
[[0, 0, 1, 1, 0, 1],[1, 0, 1, 1, 1, 0],[1, 0, 1, 1, 1, 1],[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 0]]
]
A partition for sets of sizes:  3*3  2*4 22*5
```

```
[4, 2, 2, 2, 2, 2]
[4, 0, 23]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 1, 0, 0]],
[[0, 0, 0, 1, 1, 1],[0, 0, 1, 0, 0, 1],[0, 0, 1, 1, 1, 0]],
[[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 1],[0, 0, 1, 1, 1, 1],[0, 1, 0, 0, 0, 1],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 0, 1, 0],[0, 1, 0, 0, 1, 1],[0, 1, 0, 1, 0, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 0, 0]],
[[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0],[0, 1, 1, 0, 0, 1],[1, 0, 0, 0, 1, 1],[3, 1, 0, 1, 0, 1]],
[[0, 1, 1, 1, 0, 0],[0, 1, 1, 1, 0, 1],[0, 1, 1, 1, 1, 0],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 1, 1],[1, 0, 0, 1, 0, 0],[1, 0, 0, 1, 0, 1],[2, 0, 1, 0, 1, 1]],
[[1, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[1, 0, 1, 0, 0, 0],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[2, 0, 0, 0, 1, 1],[3, 1, 0, 1, 1, 1]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 0, 0],[3, 1, 1, 0, 0, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[1, 1, 1, 1, 0, 1],[2, 0, 0, 1, 1, 0],[3, 0, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 0, 1, 0]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 1, 0, 0],[3, 0, 0, 1, 0, 1],[3, 0, 1, 0, 1, 0]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 1, 1],[3, 1, 0, 1, 1, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 1, 1, 0, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 0, 0],[3, 1, 1, 1, 1, 1]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 0, 0, 1, 0]],
[[1, 0, 1, 0, 0, 1],[2, 0, 0, 1, 1, 1],[3, 0, 0, 1, 1, 0],[3, 1, 0, 0, 0, 0],[3, 1, 1, 0, 0, 0]],
[[2, 1, 0, 0, 0, 1],[2, 1, 0, 0, 1, 0],[2, 1, 0, 1, 1, 1],[3, 0, 1, 0, 0, 0],[3, 1, 1, 1, 0, 0]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 1, 1],[1, 1, 1, 0, 0, 0],[2, 0, 1, 0, 0, 0],[3, 0, 1, 1, 0, 1]],
[[1, 1, 0, 1, 0, 0],[1, 1, 1, 0, 0, 1],[1, 1, 1, 1, 1, 0],[2, 1, 1, 1, 0, 0],[3, 0, 1, 1, 1, 1]],
[[0, 0, 0, 1, 1, 0],[0, 0, 1, 0, 1, 1],[0, 1, 1, 0, 1, 1],[2, 0, 1, 1, 0, 1],[2, 1, 1, 0, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 1, 0, 1, 1, 0],[1, 0, 1, 0, 1, 0],[1, 0, 1, 1, 1, 1],[2, 1, 0, 1, 1, 0]]
]
A partition for sets of sizes:  4*3  0*4 23*5
```

```
[4, 2, 2, 2, 2, 2]
[0, 3, 23]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 1, 0, 0],[0, 0, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0, 1],[0, 0, 0, 1, 1, 0],[0, 0, 1, 0, 0, 0],[0, 0, 1, 0, 1, 1]],
[[0, 0, 1, 0, 0, 1],[0, 0, 1, 0, 1, 0],[0, 0, 1, 1, 0, 0],[0, 0, 1, 1, 1, 1]],
[[0, 0, 1, 1, 0, 1],[0, 0, 1, 1, 1, 0],[0, 1, 0, 0, 0, 0],[1, 0, 0, 0, 0, 0],[3, 1, 0, 0, 1, 1]],
[[0, 1, 0, 0, 1, 0],[0, 1, 0, 0, 1, 1],[0, 1, 0, 1, 0, 0],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 0, 0]],
[[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0],[0, 1, 1, 0, 0, 1],[1, 0, 0, 0, 1, 1],[3, 1, 0, 1, 0, 1]],
[[0, 1, 1, 1, 0, 0],[0, 1, 1, 1, 0, 1],[0, 1, 1, 1, 1, 0],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 0, 1]],
[[0, 1, 0, 1, 0, 1],[0, 1, 1, 1, 1, 1],[1, 0, 0, 1, 0, 0],[1, 0, 0, 1, 0, 1],[2, 0, 1, 0, 1, 1]],
[[1, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[1, 0, 1, 0, 0, 0],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[2, 0, 0, 0, 1, 1],[3, 1, 0, 1, 1, 1]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 1, 0, 0],[3, 1, 1, 0, 0, 1]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[1, 1, 1, 1, 0, 1],[2, 0, 0, 1, 1, 0],[3, 0, 0, 0, 1, 0],[3, 0, 0, 0, 1, 1],[3, 1, 1, 0, 1, 0]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 1, 0, 0],[3, 0, 0, 1, 0, 1],[3, 0, 1, 0, 1, 0]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 1, 1],[3, 1, 0, 1, 1, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 1, 1, 0, 0],[3, 1, 1, 1, 1, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 0, 0],[3, 1, 1, 1, 1, 1]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 0, 0, 1, 0]],
[[1, 0, 1, 0, 0, 1],[2, 0, 0, 1, 1, 1],[3, 0, 0, 1, 1, 0],[3, 1, 0, 0, 0, 0],[3, 1, 1, 0, 0, 0]],
[[2, 1, 0, 0, 0, 1],[2, 1, 0, 0, 1, 0],[2, 1, 0, 1, 1, 1],[3, 0, 1, 0, 0, 0],[3, 1, 1, 1, 0, 0]],
[[1, 0, 1, 1, 1, 0],[1, 1, 0, 0, 1, 1],[1, 1, 1, 0, 0, 0],[2, 0, 1, 0, 0, 0],[3, 0, 1, 1, 0, 1]],
[[0, 1, 0, 0, 0, 1],[1, 1, 0, 1, 0, 0],[2, 1, 0, 1, 1, 0],[2, 1, 1, 1, 0, 0],[3, 0, 1, 1, 1, 1]],
[[0, 0, 0, 0, 1, 1],[0, 1, 1, 0, 1, 0],[1, 0, 1, 0, 1, 0],[1, 1, 1, 1, 1, 0],[2, 0, 1, 1, 0, 1]],
[[0, 1, 0, 1, 1, 0],[0, 1, 1, 0, 1, 1],[1, 0, 1, 1, 1, 1],[1, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 1]]
]
A partition for sets of sizes:  0*3  3*4 23*5
```

```
[4, 2, 2, 2, 2, 2]
[1, 1, 24]
[
[[0, 0, 0, 0, 0, 1],[0, 0, 0, 0, 1, 0],[0, 0, 0, 0, 1, 1]],
[[0, 0, 0, 1, 0, 0],[0, 0, 0, 1, 0, 1],[0, 0, 0, 1, 1, 0],[0, 0, 0, 1, 1, 1]],
[[0, 0, 1, 0, 0, 0],[0, 0, 1, 0, 0, 1],[0, 0, 1, 0, 1, 0],[0, 1, 0, 0, 0, 0],[0, 1, 1, 0, 1, 1]],
[[0, 0, 1, 1, 0, 1],[0, 0, 1, 1, 1, 0],[0, 0, 1, 1, 1, 1],[0, 1, 0, 0, 0, 1],[0, 1, 1, 1, 0, 1]],
[[0, 1, 0, 0, 1, 0],[0, 1, 0, 0, 1, 1],[0, 1, 0, 1, 0, 0],[1, 0, 0, 0, 0, 0],[3, 1, 0, 1, 0, 1]],
[[0, 1, 0, 1, 1, 1],[0, 1, 1, 0, 0, 0],[0, 1, 1, 0, 0, 1],[1, 0, 0, 0, 0, 1],[3, 1, 0, 1, 1, 1]],
[[0, 1, 1, 1, 0, 0],[0, 1, 1, 1, 1, 0],[0, 1, 1, 1, 1, 1],[1, 0, 0, 0, 1, 0],[3, 1, 1, 1, 1, 1]],
[[0, 1, 1, 0, 1, 0],[1, 0, 0, 0, 1, 1],[1, 0, 0, 1, 0, 0],[1, 0, 0, 1, 0, 1],[1, 1, 1, 0, 0, 0]],
[[1, 0, 0, 1, 1, 0],[1, 0, 0, 1, 1, 1],[1, 0, 1, 0, 0, 0],[2, 0, 0, 0, 0, 0],[3, 0, 1, 0, 0, 1]],
[[1, 0, 1, 0, 1, 1],[1, 0, 1, 1, 0, 0],[1, 0, 1, 1, 0, 1],[2, 0, 0, 0, 0, 1],[3, 0, 1, 0, 1, 1]],
[[1, 1, 0, 0, 0, 0],[1, 1, 0, 0, 0, 1],[1, 1, 0, 0, 1, 0],[2, 0, 0, 0, 1, 0],[3, 1, 0, 0, 0, 1]],
[[1, 1, 0, 1, 0, 1],[1, 1, 0, 1, 1, 0],[1, 1, 0, 1, 1, 1],[2, 0, 0, 1, 0, 0],[3, 1, 0, 0, 0, 0]],
[[1, 1, 1, 0, 1, 0],[1, 1, 1, 0, 1, 1],[1, 1, 1, 1, 0, 0],[2, 0, 0, 0, 1, 1],[3, 1, 1, 1, 1, 0]],
[[1, 1, 1, 1, 1, 1],[2, 0, 0, 1, 0, 1],[3, 0, 0, 0, 0, 0],[3, 0, 0, 0, 0, 1],[3, 1, 1, 0, 1, 1]],
[[0, 0, 1, 0, 1, 1],[1, 0, 1, 0, 0, 1],[2, 0, 0, 1, 1, 0],[2, 0, 0, 1, 1, 1],[3, 0, 0, 0, 1, 1]],
[[2, 0, 1, 0, 0, 1],[2, 0, 1, 0, 1, 0],[2, 0, 1, 0, 1, 1],[3, 0, 0, 0, 1, 0],[3, 0, 1, 0, 1, 0]],
[[2, 0, 1, 1, 1, 0],[2, 0, 1, 1, 1, 1],[2, 1, 0, 0, 0, 0],[3, 0, 0, 1, 0, 1],[3, 1, 0, 1, 0, 0]],
[[2, 1, 0, 0, 1, 1],[2, 1, 0, 1, 0, 0],[2, 1, 0, 1, 0, 1],[3, 0, 0, 1, 0, 0],[3, 1, 0, 1, 1, 0]],
[[2, 1, 1, 0, 0, 0],[2, 1, 1, 0, 0, 1],[2, 1, 1, 0, 1, 0],[3, 0, 0, 1, 1, 0],[3, 1, 1, 1, 0, 1]],
[[2, 1, 1, 1, 0, 1],[2, 1, 1, 1, 1, 0],[2, 1, 1, 1, 1, 1],[3, 0, 1, 1, 1, 0],[3, 1, 0, 0, 1, 0]],
[[2, 0, 1, 1, 0, 0],[2, 1, 0, 1, 1, 0],[2, 1, 1, 0, 1, 1],[3, 0, 1, 1, 0, 0],[3, 0, 1, 1, 0, 1]],
[[1, 0, 1, 0, 1, 0],[2, 0, 1, 1, 0, 1],[3, 0, 1, 0, 0, 0],[3, 1, 0, 0, 1, 1],[3, 1, 1, 1, 0, 0]],
[[2, 1, 0, 0, 0, 1],[2, 1, 0, 0, 1, 0],[2, 1, 1, 1, 0, 0],[3, 0, 0, 1, 1, 1],[3, 1, 1, 0, 0, 0]],
[[0, 1, 0, 1, 1, 0],[1, 1, 0, 0, 1, 1],[2, 0, 1, 0, 0, 0],[2, 1, 0, 1, 1, 1],[3, 1, 1, 0, 1, 0]],
[[0, 0, 1, 1, 0, 0],[1, 0, 1, 1, 1, 0],[1, 1, 0, 1, 0, 0],[3, 0, 1, 1, 1, 1],[3, 1, 1, 0, 0, 1]],
[[0, 1, 0, 1, 0, 1],[1, 0, 1, 1, 1, 1],[1, 1, 1, 0, 0, 1],[1, 1, 1, 1, 0, 1],[1, 1, 1, 1, 1, 0]]
]
A partition for sets of sizes:  1*3  1*4 24*5
```

E Zero sum partitions of $\left((\mathbb{Z}_4)^2 \times \mathbb{Z}_2\right)^*$ (with $b \leq 3$)

```
[4, 4, 2]
[7, 0, 2]
[
[[0, 0, 1],[0, 1, 0],[0, 3, 1]],
[[0, 2, 0],[1, 0, 0],[3, 2, 0]],
[[0, 1, 1],[1, 0, 1],[3, 3, 0]],
[[1, 1, 0],[1, 1, 1],[2, 2, 1]],
[[1, 2, 1],[1, 3, 0],[2, 3, 1]],
[[2, 0, 0],[3, 1, 1],[3, 3, 1]],
[[2, 1, 1],[3, 1, 0],[3, 2, 1]],
[[2, 0, 1],[2, 1, 0],[2, 3, 0],[3, 0, 0],[3, 0, 1]],
[[0, 2, 1],[0, 3, 0],[1, 2, 0],[1, 3, 1],[2, 2, 0]]
]
A partition for sets of sizes:  7*3  0*4  2*5
```

```
[4, 4, 2]
[2, 0, 5]
[
[[0, 0, 1],[0, 1, 0],[0, 3, 1]],
[[0, 2, 0],[1, 0, 0],[3, 2, 0]],
[[0, 1, 1],[0, 2, 1],[1, 0, 1],[1, 1, 0],[2, 0, 1]],
[[1, 2, 0],[1, 2, 1],[1, 3, 0],[2, 0, 0],[3, 1, 1]],
[[1, 1, 1],[2, 1, 0],[3, 0, 0],[3, 3, 0],[3, 3, 1]],
[[2, 1, 1],[2, 2, 0],[2, 3, 1],[3, 0, 1],[3, 2, 1]],
[[0, 3, 0],[1, 3, 1],[2, 2, 1],[2, 3, 0],[3, 1, 0]]
]
A partition for sets of sizes:  2*3  0*4  5*5
```

F Zero sum partitions of $\left((\mathbb{Z}_4)^3\right)^*$ (with $b \leq 3$)

```
[4, 4, 4]
[21, 0, 0]
[
[[0, 0, 1],[0, 1, 0],[0, 3, 3]],
[[0, 0, 2],[0, 1, 1],[0, 3, 1]],
[[0, 1, 3],[1, 0, 0],[3, 3, 1]],
[[0, 2, 2],[0, 3, 0],[0, 3, 2]],
[[0, 1, 2],[1, 0, 2],[3, 3, 0]],
[[0, 2, 0],[1, 0, 1],[3, 2, 3]],
[[1, 0, 3],[1, 1, 0],[2, 3, 1]],
[[1, 1, 2],[1, 1, 3],[2, 2, 3]],
[[1, 2, 1],[1, 2, 2],[2, 0, 1]],
[[1, 3, 0],[1, 3, 2],[2, 2, 2]],
[[2, 0, 0],[3, 0, 1],[3, 0, 3]],
[[2, 0, 2],[3, 0, 0],[3, 0, 2]],
[[2, 1, 1],[3, 1, 1],[3, 2, 2]],
[[2, 2, 1],[3, 1, 0],[3, 1, 3]],
[[1, 2, 0],[1, 3, 1],[2, 3, 3]],
[[2, 3, 2],[3, 2, 0],[3, 3, 2]],
[[2, 3, 0],[3, 2, 1],[3, 3, 3]],
[[1, 1, 1],[1, 2, 3],[2, 1, 0]],
[[0, 2, 3],[2, 1, 2],[2, 1, 3]],
[[0, 0, 3],[1, 3, 3],[3, 1, 2]],
[[0, 2, 1],[2, 0, 3],[2, 2, 0]]
]
A partition for sets of sizes: 21*3  0*4  0*5
```

```
[4, 4, 4]
[16, 0, 3]
[
[[0, 0, 1],[0, 1, 0],[0, 3, 3]],
[[0, 0, 2],[0, 1, 1],[0, 3, 1]],
[[0, 1, 3],[1, 0, 0],[3, 3, 1]],
[[0, 2, 2],[0, 3, 0],[0, 3, 2]],
[[0, 1, 2],[1, 0, 2],[3, 3, 0]],
[[0, 2, 0],[1, 0, 1],[3, 2, 3]],
[[1, 0, 3],[1, 1, 0],[2, 3, 1]],
[[1, 1, 2],[1, 1, 3],[2, 2, 3]],
[[1, 2, 1],[1, 2, 2],[2, 0, 1]],
[[1, 3, 0],[1, 3, 2],[2, 2, 2]],
[[2, 0, 0],[3, 0, 1],[3, 0, 3]],
[[2, 0, 2],[3, 0, 0],[3, 0, 2]],
[[2, 1, 1],[3, 1, 1],[3, 2, 2]],
[[2, 2, 1],[3, 1, 0],[3, 1, 3]],
[[1, 2, 0],[1, 3, 1],[2, 3, 3]],
[[2, 3, 2],[3, 2, 0],[3, 3, 2]],
[[2, 1, 0],[2, 1, 3],[2, 2, 0],[3, 1, 2],[3, 3, 3]],
[[0, 2, 3],[1, 1, 1],[2, 0, 3],[2, 3, 0],[3, 2, 1]],
[[0, 0, 3],[0, 2, 1],[1, 2, 3],[1, 3, 3],[2, 1, 2]]
]
A partition for sets of sizes: 16*3  0*4  3*5
```

```
[4, 4, 4]
[11, 0, 6]
[
[[0, 0, 1],[0, 1, 0],[0, 3, 3]],
[[0, 0, 2],[0, 1, 1],[0, 3, 1]],
[[0, 1, 3],[1, 0, 0],[3, 3, 1]],
[[0, 2, 2],[0, 3, 0],[0, 3, 2]],
[[0, 1, 2],[1, 0, 2],[3, 3, 0]],
[[0, 2, 0],[1, 0, 1],[3, 2, 3]],
[[1, 0, 3],[1, 1, 0],[2, 3, 1]],
[[1, 1, 2],[1, 1, 3],[2, 2, 3]],
[[1, 2, 1],[1, 2, 2],[2, 0, 1]],
[[1, 3, 0],[1, 3, 2],[2, 2, 2]],
[[2, 0, 0],[3, 0, 1],[3, 0, 3]],
[[2, 0, 2],[2, 0, 3],[2, 1, 0],[3, 0, 0],[3, 3, 3]],
[[2, 1, 3],[2, 2, 0],[2, 2, 1],[3, 0, 2],[3, 3, 2]],
[[2, 3, 0],[2, 3, 2],[2, 3, 3],[3, 1, 1],[3, 2, 2]],
[[1, 3, 3],[2, 1, 1],[3, 1, 0],[3, 1, 3],[3, 2, 1]],
[[0, 0, 3],[1, 1, 1],[1, 2, 0],[1, 2, 3],[1, 3, 1]],
[[0, 2, 1],[0, 2, 3],[2, 1, 2],[3, 1, 2],[3, 2, 0]]
]
A partition for sets of sizes: 11*3  0*4  6*5

[4, 4, 4]
[6, 0, 9]
[
[[0, 0, 1],[0, 1, 0],[0, 3, 3]],
[[0, 0, 2],[0, 1, 1],[0, 3, 1]],
[[0, 1, 3],[1, 0, 0],[3, 3, 1]],
[[0, 2, 2],[0, 3, 0],[0, 3, 2]],
[[0, 1, 2],[1, 0, 2],[3, 3, 0]],
[[0, 2, 0],[1, 0, 1],[3, 2, 3]],
[[1, 0, 3],[1, 1, 0],[1, 1, 1],[2, 0, 0],[3, 2, 0]],
[[1, 2, 0],[1, 2, 1],[1, 2, 2],[2, 0, 3],[3, 2, 2]],
[[1, 3, 1],[1, 3, 2],[1, 3, 3],[2, 1, 1],[3, 2, 1]],
[[2, 0, 2],[2, 1, 0],[2, 1, 2],[3, 1, 1],[3, 1, 3]],
[[2, 1, 3],[2, 2, 0],[2, 2, 1],[3, 0, 1],[3, 3, 3]],
[[2, 3, 0],[2, 3, 1],[2, 3, 2],[3, 0, 3],[3, 3, 2]],
[[2, 2, 2],[2, 2, 3],[2, 3, 3],[3, 0, 0],[3, 1, 0]],
[[0, 0, 3],[0, 2, 3],[1, 1, 2],[1, 1, 3],[2, 0, 1]],
[[0, 2, 1],[1, 2, 3],[1, 3, 0],[3, 0, 2],[3, 1, 2]]
]
A partition for sets of sizes:  6*3  0*4  9*5

[4, 4, 4]
[1, 0, 12]
[
[[0, 0, 1],[0, 1, 0],[0, 3, 3]],
[[0, 0, 2],[0, 1, 1],[0, 1, 2],[0, 3, 1],[0, 3, 2]],
[[0, 2, 1],[0, 2, 2],[0, 2, 3],[1, 0, 0],[3, 2, 2]],
[[0, 2, 0],[0, 3, 0],[1, 0, 1],[1, 0, 2],[2, 3, 1]],
[[1, 0, 3],[1, 1, 0],[1, 1, 1],[2, 0, 0],[3, 2, 0]],
[[1, 2, 0],[1, 2, 1],[1, 2, 2],[2, 0, 2],[3, 2, 3]],
[[1, 3, 1],[1, 3, 2],[1, 3, 3],[2, 0, 1],[3, 3, 1]],
[[1, 2, 3],[2, 0, 3],[3, 0, 0],[3, 0, 1],[3, 2, 1]],
[[2, 1, 3],[2, 2, 0],[2, 2, 1],[3, 0, 2],[3, 3, 2]],
[[2, 3, 0],[2, 3, 2],[2, 3, 3],[3, 0, 3],[3, 3, 0]],
[[2, 1, 1],[2, 1, 2],[2, 2, 2],[3, 1, 0],[3, 3, 3]],
[[0, 0, 3],[1, 1, 2],[1, 1, 3],[3, 1, 1],[3, 1, 3]],
[[0, 1, 3],[1, 3, 0],[2, 1, 0],[2, 2, 3],[3, 1, 2]]
]
A partition for sets of sizes:  1*3  0*4 12*5
```

G Zero sum partitions of $\left(\mathbb{Z}_8 \times (\mathbb{Z}_2)^2\right)^*$ (with $b \leq 11$)

```
[8, 2, 2]
[9, 1, 0]
[
[[0, 0, 1],[0, 1, 0],[0, 1, 1]],
[[1, 0, 0],[1, 0, 1],[6, 0, 1]],
[[1, 1, 1],[2, 0, 0],[5, 1, 1]],
[[2, 1, 0],[2, 1, 1],[4, 0, 1]],
[[3, 0, 1],[6, 0, 0],[7, 0, 1]],
[[4, 0, 0],[5, 0, 0],[7, 0, 0]],
[[4, 1, 1],[5, 0, 1],[7, 1, 0]],
[[2, 0, 1],[3, 1, 0],[3, 1, 1]],
[[1, 1, 0],[3, 0, 0],[4, 1, 0]],
[[5, 1, 0],[6, 1, 0],[6, 1, 1],[7, 1, 1]]
]
A partition for sets of sizes:  9*3  1*4  0*5

[8, 2, 2]
[6, 2, 1]
[
[[0, 0, 1],[0, 1, 0],[0, 1, 1]],
[[1, 0, 0],[1, 0, 1],[6, 0, 1]],
[[1, 1, 1],[2, 0, 0],[5, 1, 1]],
[[2, 1, 0],[2, 1, 1],[4, 0, 1]],
[[3, 0, 1],[6, 0, 0],[7, 0, 1]],
[[4, 0, 0],[5, 0, 0],[7, 0, 0]],
[[4, 1, 1],[6, 1, 0],[7, 1, 0],[7, 1, 1]],
[[2, 0, 1],[3, 0, 0],[5, 1, 0],[6, 1, 1]],
[[1, 1, 0],[3, 1, 0],[3, 1, 1],[4, 1, 0],[5, 0, 1]]
]
A partition for sets of sizes:  6*3  2*4  1*5

[8, 2, 2]
[7, 0, 2]
[
[[0, 0, 1],[0, 1, 0],[0, 1, 1]],
[[1, 0, 0],[1, 0, 1],[6, 0, 1]],
[[1, 1, 1],[2, 0, 0],[5, 1, 1]],
[[2, 1, 0],[2, 1, 1],[4, 0, 1]],
[[3, 0, 1],[6, 0, 0],[7, 0, 1]],
[[4, 0, 0],[5, 0, 0],[7, 0, 0]],
[[4, 1, 1],[5, 0, 1],[7, 1, 0]],
[[2, 0, 1],[4, 1, 0],[5, 1, 0],[6, 1, 0],[7, 1, 1]],
[[1, 1, 0],[3, 0, 0],[3, 1, 0],[3, 1, 1],[6, 1, 1]]
]
A partition for sets of sizes:  7*3  0*4  2*5

[8, 2, 2]
[4, 1, 3]
[
[[0, 0, 1],[0, 1, 0],[0, 1, 1]],
[[1, 0, 0],[1, 0, 1],[6, 0, 1]],
[[1, 1, 1],[2, 0, 0],[5, 1, 1]],
[[2, 1, 0],[2, 1, 1],[4, 0, 1]],
[[3, 0, 1],[3, 1, 0],[3, 1, 1],[7, 0, 0]],
[[3, 0, 0],[4, 1, 0],[4, 1, 1],[6, 0, 0],[7, 0, 1]],
[[2, 0, 1],[5, 0, 0],[5, 0, 1],[5, 1, 0],[7, 1, 0]],
[[1, 1, 0],[4, 0, 0],[6, 1, 0],[6, 1, 1],[7, 1, 1]]
]
A partition for sets of sizes:  4*3  1*4  3*5
```

```
[8, 2, 2]
[1, 2, 4]
[
[[0, 0, 1],[0, 1, 0],[0, 1, 1]],
[[1, 0, 0],[1, 0, 1],[1, 1, 0],[5, 1, 1]],
[[2, 0, 0],[2, 0, 1],[2, 1, 0],[2, 1, 1]],
[[3, 0, 0],[3, 0, 1],[3, 1, 0],[3, 1, 1],[4, 0, 0]],
[[4, 0, 1],[4, 1, 0],[4, 1, 1],[5, 0, 0],[7, 0, 0]],
[[5, 1, 0],[6, 1, 0],[7, 0, 1],[7, 1, 0],[7, 1, 1]],
[[1, 1, 1],[5, 0, 1],[6, 0, 0],[6, 0, 1],[6, 1, 1]]
]
A partition for sets of sizes:  1*3  2*4  4*5

[8, 2, 2]
[2, 0, 5]
[
[[0, 0, 1],[0, 1, 0],[0, 1, 1]],
[[1, 0, 0],[1, 0, 1],[6, 0, 1]],
[[1, 1, 1],[2, 0, 0],[2, 0, 1],[4, 0, 0],[7, 1, 0]],
[[3, 0, 0],[3, 0, 1],[4, 1, 0],[7, 0, 0],[7, 1, 1]],
[[4, 0, 1],[4, 1, 1],[5, 0, 0],[5, 0, 1],[6, 1, 1]],
[[1, 1, 0],[5, 1, 0],[5, 1, 1],[6, 1, 0],[7, 0, 1]],
[[2, 1, 0],[2, 1, 1],[3, 1, 0],[3, 1, 1],[6, 0, 0]]
]
A partition for sets of sizes:  2*3  0*4  5*5
```

H Zero sum partitions of $\left(\mathbb{Z}_4 \times (\mathbb{Z}_2)^2\right)^* + \left((\mathbb{Z}_2)^2\right)^*$

```
[4, 2, 2, 2, 2]
[15, 0, 0]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 1],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[2, 0, 0, 1, 0],[3, 0, 1, 0, 1],[3, 0, 1, 1, 1]],
[[2, 0, 1, 1, 1],[3, 0, 1, 1, 0],[3, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 0, 1],[3, 0, 0, 1, 1]],
[[0, 0, 0, 1, 1],[2, 0, 1, 1, 0],[2, 0, 1, 0, 1]],
[[0, 0, 1, 1, 1],[2, 0, 0, 0, 1],[2, 0, 1, 1, 0]],
[[2, 0, 0, 1, 1],[3, 0, 0, 0, 1],[3, 0, 0, 1, 0]]
]
A partition for sets of sizes: 15*3  0*4  0*5

[4, 2, 2, 2, 2]
[11, 3, 0]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 1],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[2, 0, 0, 0, 1],[3, 0, 0, 1, 1],[3, 0, 0, 1, 0]],
[[2, 0, 1, 1, 0],[3, 0, 0, 0, 1],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 0],[2, 0, 1, 0, 1],[3, 0, 1, 0, 1],[3, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1],[2, 0, 0, 1, 1],[2, 0, 1, 1, 1],[3, 0, 0, 0, 1]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[2, 0, 1, 1, 0],[2, 0, 0, 1, 0]]
]
A partition for sets of sizes: 11*3  3*4  0*5

[4, 2, 2, 2, 2]
[7, 6, 0]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1],[2, 0, 1, 0, 1],[2, 0, 0, 0, 1],[3, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[2, 0, 1, 1, 0],[3, 0, 0, 1, 0],[3, 0, 0, 1, 0]],
[[0, 0, 1, 1, 0],[2, 0, 1, 0, 1],[3, 0, 1, 1, 0],[3, 0, 1, 0, 1]],
[[1, 0, 0, 1, 1],[2, 0, 0, 1, 0],[2, 0, 1, 1, 0],[3, 0, 1, 1, 1]],
[[0, 0, 0, 1, 1],[2, 0, 1, 1, 1],[3, 0, 0, 1, 1],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 1],[2, 0, 0, 1, 1],[3, 0, 1, 0, 1],[3, 0, 0, 0, 1]]
]
A partition for sets of sizes:  7*3  6*4  0*5
```

```
[4, 2, 2, 2, 2]
[3, 9, 0]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 0, 1],[1, 0, 0, 0, 1],[1, 0, 1, 0, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 1, 1, 1]],
[[2, 0, 0, 0, 1],[2, 0, 0, 1, 0],[2, 0, 1, 0, 1],[2, 0, 1, 1, 0]],
[[2, 0, 0, 1, 1],[2, 0, 1, 1, 1],[2, 0, 0, 0, 1],[2, 0, 1, 0, 1]],
[[0, 0, 1, 1, 0],[2, 0, 0, 1, 0],[3, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[3, 0, 0, 1, 0],[3, 0, 0, 1, 1],[3, 0, 1, 1, 0],[3, 0, 1, 1, 1]],
[[0, 0, 0, 1, 1],[2, 0, 1, 1, 0],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 1],[2, 0, 0, 1, 1],[3, 0, 1, 0, 1],[3, 0, 0, 0, 1]]
]
A partition for sets of sizes:  3*3  9*4  0*5

[4, 2, 2, 2, 2]
[12, 1, 1]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 1],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[2, 0, 0, 0, 1],[3, 0, 0, 1, 1],[3, 0, 0, 1, 0]],
[[2, 0, 1, 1, 0],[3, 0, 0, 0, 1],[3, 0, 1, 1, 1]],
[[0, 0, 0, 1, 1],[2, 0, 1, 1, 0],[2, 0, 1, 0, 1]],
[[1, 0, 1, 0, 1],[2, 0, 0, 1, 1],[2, 0, 1, 1, 1],[3, 0, 0, 0, 1]],
[[0, 0, 1, 1, 1],[0, 0, 1, 1, 0],[2, 0, 0, 1, 0],[3, 0, 1, 0, 1],[3, 0, 1, 1, 0]]
]
A partition for sets of sizes: 12*3  1*4  1*5

[4, 2, 2, 2, 2]
[8, 4, 1]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 1],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 0, 0, 1],[2, 0, 0, 1, 0]],
[[0, 0, 1, 1, 0],[2, 0, 0, 1, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[3, 0, 0, 1, 1],[3, 0, 1, 0, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[1, 0, 1, 0, 1],[3, 0, 0, 0, 1]],
[[2, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[3, 0, 1, 1, 0],[3, 0, 0, 1, 0]]
]
A partition for sets of sizes:  8*3  4*4  1*5
```

```
[4, 2, 2, 2, 2]
[4, 7, 1]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[1, 0, 0, 0, 1],[1, 0, 0, 1, 1]],
[[1, 0, 0, 1, 0],[1, 0, 1, 0, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1],[2, 0, 0, 0, 1],[2, 0, 0, 1, 1],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 0, 0, 1],[2, 0, 0, 1, 0]],
[[0, 0, 1, 1, 0],[2, 0, 0, 1, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 0],[2, 0, 1, 1, 1],[3, 0, 0, 1, 1],[3, 0, 0, 1, 0]],
[[0, 0, 0, 1, 1],[2, 0, 1, 0, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 1],[1, 0, 1, 1, 0],[2, 0, 0, 1, 0],[2, 0, 1, 1, 0],[3, 0, 1, 0, 1]]
]
A partition for sets of sizes:  4*3  7*4  1*5
```

```
[4, 2, 2, 2, 2]
[0, 10, 1]
[
[[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[0, 0, 0, 0, 1],[0, 0, 0, 1, 0]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[1, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[0, 0, 1, 1, 1],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1],[2, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 0],[1, 0, 0, 0, 1],[1, 0, 0, 1, 0]],
[[1, 0, 1, 1, 1],[1, 0, 0, 1, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1],[2, 0, 0, 0, 1],[2, 0, 1, 0, 1],[3, 0, 0, 1, 1]],
[[2, 0, 0, 1, 0],[2, 0, 1, 1, 0],[2, 0, 0, 0, 1],[2, 0, 1, 0, 1]],
[[0, 0, 1, 0, 1],[2, 0, 0, 1, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[2, 0, 1, 1, 1],[2, 0, 1, 1, 1],[3, 0, 1, 0, 1]],
[[3, 0, 0, 1, 1],[3, 0, 1, 1, 0],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 1, 0],[2, 0, 0, 1, 1],[2, 0, 0, 1, 0],[3, 0, 0, 0, 1]]
]
A partition for sets of sizes:  0*3 10*4  1*5
```

```
[4, 2, 2, 2, 2]
[9, 2, 2]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 1],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[2, 0, 0, 0, 1],[2, 0, 0, 1, 0],[2, 0, 1, 0, 1],[2, 0, 1, 1, 0]],
[[3, 0, 0, 1, 1],[3, 0, 1, 0, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[3, 0, 0, 1, 0]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[2, 0, 0, 1, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 0]]
]
A partition for sets of sizes:  9*3  2*4  2*5
```

```
[4, 2, 2, 2, 2]
[5, 5, 2]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[1, 0, 1, 0, 1],[1, 0, 1, 1, 0]],
[[1, 0, 0, 1, 1],[1, 0, 1, 1, 1],[3, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[2, 0, 0, 1, 1],[2, 0, 1, 0, 1],[2, 0, 0, 0, 1],[2, 0, 1, 1, 1]],
[[0, 0, 1, 1, 0],[2, 0, 0, 1, 0],[3, 0, 0, 1, 0],[3, 0, 1, 1, 0]],
[[3, 0, 0, 1, 1],[3, 0, 1, 1, 1],[3, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[0, 0, 1, 1, 0],[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[2, 0, 0, 0, 1],[2, 0, 0, 1, 1]],
[[2, 0, 1, 1, 0],[2, 0, 1, 0, 1],[2, 0, 1, 1, 0],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]]
]
A partition for sets of sizes:  5*3  5*4  2*5

[4, 2, 2, 2, 2]
[1, 8, 2]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 1],[1, 0, 0, 0, 1],[3, 0, 0, 1, 1]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1],[2, 0, 1, 1, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 0, 1]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[1, 0, 1, 0, 1],[1, 0, 1, 1, 0]],
[[1, 0, 0, 1, 1],[1, 0, 1, 1, 1],[3, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[2, 0, 0, 1, 1],[2, 0, 1, 1, 0],[2, 0, 0, 1, 1],[2, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1],[2, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 0, 0, 1]],
[[0, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 0, 1, 1],[3, 0, 1, 0, 1]],
[[0, 0, 1, 0, 1],[3, 0, 1, 1, 0],[3, 0, 0, 1, 0],[3, 0, 1, 1, 0],[3, 0, 1, 1, 1]],
[[2, 0, 0, 1, 0],[2, 0, 0, 1, 0],[2, 0, 1, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]]
]
A partition for sets of sizes:  1*3  8*4  2*5

[4, 2, 2, 2, 2]
[10, 0, 3]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 1],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[2, 0, 0, 0, 1],[3, 0, 0, 1, 1],[3, 0, 0, 1, 0]],
[[2, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[3, 0, 1, 0, 1],[3, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[2, 0, 0, 1, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 0]]
]
A partition for sets of sizes: 10*3  0*4  3*5
```

```
[4, 2, 2, 2, 2]
[6, 3, 3]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 0],[3, 0, 0, 0, 1],[3, 0, 0, 1, 0]],
[[0, 0, 1, 1, 0],[2, 0, 1, 0, 1],[3, 0, 1, 0, 1],[3, 0, 1, 1, 0]],
[[0, 0, 1, 1, 0],[2, 0, 0, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[3, 0, 0, 1, 1],[3, 0, 1, 1, 1]],
[[0, 0, 0, 1, 1],[1, 0, 0, 1, 1],[2, 0, 0, 0, 1],[2, 0, 1, 1, 0],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0],[2, 0, 0, 1, 1],[3, 0, 0, 0, 1]]
]
A partition for sets of sizes:  6*3  3*4  3*5

[4, 2, 2, 2, 2]
[2, 6, 3]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[1, 0, 0, 0, 1],[3, 0, 0, 1, 0]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 1],[1, 0, 1, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 1, 1],[1, 0, 0, 0, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 0],[3, 0, 1, 0, 1],[3, 0, 1, 1, 0]],
[[2, 0, 0, 1, 0],[2, 0, 1, 0, 1],[2, 0, 0, 0, 1],[2, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1],[2, 0, 1, 1, 1],[2, 0, 0, 1, 1],[3, 0, 0, 0, 1]],
[[2, 0, 1, 1, 0],[2, 0, 1, 0, 1],[2, 0, 1, 1, 1],[3, 0, 0, 1, 1],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 1],[1, 0, 0, 1, 1],[1, 0, 1, 1, 1],[3, 0, 1, 0, 1],[3, 0, 1, 1, 0]],
[[0, 0, 0, 1, 1],[1, 0, 0, 1, 0],[2, 0, 0, 0, 1],[2, 0, 0, 1, 0],[3, 0, 0, 1, 0]]
]
A partition for sets of sizes:  2*3  6*4  3*5

[4, 2, 2, 2, 2]
[7, 1, 4]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 0, 0, 1]],
[[1, 0, 1, 0, 1],[2, 0, 1, 0, 1],[2, 0, 0, 0, 1],[3, 0, 0, 0, 1]],
[[2, 0, 1, 1, 0],[2, 0, 0, 1, 0],[2, 0, 0, 1, 1],[3, 0, 0, 1, 0],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[3, 0, 0, 1, 1],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 0],[3, 0, 1, 1, 0],[3, 0, 1, 1, 1],[3, 0, 0, 1, 0],[3, 0, 1, 0, 1]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[0, 0, 1, 1, 0],[1, 0, 0, 1, 1],[3, 0, 0, 0, 1]]
]
A partition for sets of sizes:  7*3  1*4  4*5
```

```
[4, 2, 2, 2, 2]
[3, 4, 4]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 0, 1],[1, 0, 0, 0, 1],[1, 0, 1, 0, 1]],
[[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 1, 0],[1, 0, 1, 1, 1],[2, 0, 1, 1, 1]],
[[2, 0, 0, 0, 1],[2, 0, 0, 1, 0],[2, 0, 1, 0, 1],[2, 0, 1, 1, 0]],
[[2, 0, 0, 1, 1],[2, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[0, 0, 1, 1, 0],[3, 0, 0, 0, 1],[3, 0, 0, 1, 1],[3, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[2, 0, 0, 1, 0],[2, 0, 0, 1, 1],[2, 0, 1, 1, 0],[3, 0, 1, 0, 1],[3, 0, 0, 1, 0]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[2, 0, 1, 0, 1],[3, 0, 1, 1, 0],[3, 0, 1, 1, 1]]
]
A partition for sets of sizes:  3*3  4*4  4*5

[4, 2, 2, 2, 2]
[4, 2, 5]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[1, 0, 0, 0, 1],[1, 0, 0, 1, 1]],
[[1, 0, 0, 1, 0],[1, 0, 1, 0, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 0, 1, 0],[3, 0, 0, 1, 1],[3, 0, 1, 1, 1]],
[[2, 0, 1, 1, 0],[2, 0, 0, 0, 1],[2, 0, 0, 1, 1],[3, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[0, 0, 1, 1, 0],[2, 0, 0, 1, 0],[2, 0, 0, 1, 1],[2, 0, 0, 1, 0],[2, 0, 1, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 1, 0],[2, 0, 1, 0, 1],[2, 0, 1, 1, 1],[3, 0, 0, 1, 0]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[2, 0, 1, 1, 0],[3, 0, 1, 0, 1],[3, 0, 1, 1, 1]]
]
A partition for sets of sizes:  4*3  2*4  5*5

[4, 2, 2, 2, 2]
[0, 5, 5]
[
[[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[0, 0, 0, 0, 1],[0, 0, 0, 1, 0]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[1, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[0, 0, 1, 1, 1],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1],[2, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 0],[1, 0, 0, 0, 1],[1, 0, 0, 1, 0]],
[[1, 0, 1, 1, 1],[1, 0, 0, 1, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 0]],
[[1, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 0, 0, 1],[3, 0, 1, 0, 1],[3, 0, 0, 1, 0]],
[[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 0, 1, 1],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 1, 0, 1],[2, 0, 0, 1, 0],[2, 0, 1, 0, 1],[3, 0, 1, 1, 1]],
[[2, 0, 0, 1, 0],[2, 0, 0, 1, 1],[2, 0, 1, 1, 1],[3, 0, 1, 1, 1],[3, 0, 0, 0, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 1, 0],[2, 0, 0, 1, 1],[2, 0, 1, 0, 1],[3, 0, 1, 1, 0]]
]
A partition for sets of sizes:  0*3  5*4  5*5
```

```
[4, 2, 2, 2, 2]
[5, 0, 6]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[2, 0, 0, 1, 0]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1],[2, 0, 0, 0, 1],[3, 0, 0, 0, 1]],
[[1, 0, 1, 1, 1],[2, 0, 0, 1, 1],[3, 0, 0, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[2, 0, 1, 1, 0],[2, 0, 0, 0, 1],[2, 0, 0, 1, 0],[3, 0, 0, 1, 1],[3, 0, 1, 1, 0]],
[[0, 0, 1, 1, 0],[1, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[3, 0, 0, 1, 0]],
[[0, 0, 0, 1, 1],[1, 0, 1, 1, 0],[2, 0, 1, 0, 1],[2, 0, 1, 0, 1],[3, 0, 1, 0, 1]],
[[0, 0, 1, 1, 1],[0, 0, 1, 1, 0],[2, 0, 0, 1, 1],[3, 0, 1, 0, 1],[3, 0, 1, 1, 1]]
]
A partition for sets of sizes:  5*3  0*4  6*5

[4, 2, 2, 2, 2]
[1, 3, 6]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[0, 0, 0, 1, 1],[1, 0, 0, 0, 1],[3, 0, 0, 1, 1]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1],[2, 0, 1, 1, 1]],
[[0, 0, 1, 1, 0],[1, 0, 1, 0, 1],[1, 0, 1, 1, 0],[2, 0, 1, 0, 1]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1],[2, 0, 0, 0, 1],[3, 0, 0, 0, 1]],
[[1, 0, 1, 1, 1],[2, 0, 0, 1, 0],[3, 0, 0, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 0]],
[[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 1, 0, 1],[3, 0, 1, 0, 1]],
[[0, 0, 1, 1, 1],[1, 0, 1, 0, 1],[2, 0, 0, 1, 1],[2, 0, 1, 1, 0],[3, 0, 1, 1, 1]],
[[0, 0, 1, 0, 1],[1, 0, 1, 1, 1],[1, 0, 1, 1, 0],[3, 0, 0, 1, 1],[3, 0, 1, 1, 1]],
[[2, 0, 0, 1, 0],[2, 0, 0, 1, 1],[2, 0, 1, 0, 1],[3, 0, 1, 1, 0],[3, 0, 0, 1, 0]]
]
A partition for sets of sizes:  1*3  3*4  6*5

[4, 2, 2, 2, 2]
[2, 1, 7]
[
[[0, 0, 1, 0, 1],[0, 0, 0, 1, 0],[0, 0, 1, 1, 1]],
[[0, 0, 0, 0, 1],[1, 0, 0, 1, 0],[3, 0, 0, 1, 1]],
[[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[1, 0, 0, 0, 1],[3, 0, 0, 1, 0]],
[[0, 0, 1, 1, 0],[1, 0, 0, 1, 1],[1, 0, 1, 1, 0],[1, 0, 0, 0, 1],[1, 0, 0, 1, 0]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 1],[1, 0, 0, 1, 1],[2, 0, 0, 1, 1],[3, 0, 0, 1, 0]],
[[1, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 0, 0, 1],[3, 0, 1, 1, 0],[3, 0, 0, 0, 1]],
[[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 1, 0, 1],[3, 0, 1, 0, 1]],
[[2, 0, 1, 0, 1],[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[3, 0, 0, 1, 1],[3, 0, 1, 1, 1]],
[[1, 0, 1, 0, 1],[1, 0, 1, 1, 0],[2, 0, 0, 1, 0],[2, 0, 0, 1, 0],[2, 0, 0, 1, 1]],
[[0, 0, 1, 1, 1],[0, 0, 0, 1, 1],[2, 0, 1, 0, 1],[3, 0, 1, 1, 1],[3, 0, 1, 1, 0]]
]
A partition for sets of sizes:  2*3  1*4  7*5

[4, 2, 2, 2, 2]
[0, 0, 9]
[
[[0, 0, 1, 0, 1],[0, 0, 1, 1, 0],[0, 0, 1, 1, 1],[0, 0, 0, 0, 1],[0, 0, 1, 0, 1]],
[[0, 0, 0, 1, 1],[0, 0, 1, 1, 0],[0, 0, 1, 1, 1],[1, 0, 0, 0, 1],[3, 0, 0, 1, 1]],
[[1, 0, 0, 1, 0],[1, 0, 0, 1, 1],[1, 0, 1, 0, 1],[2, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[1, 0, 0, 0, 1],[1, 0, 0, 1, 0],[1, 0, 0, 1, 1],[2, 0, 0, 1, 0],[3, 0, 0, 1, 0]],
[[1, 0, 1, 1, 1],[2, 0, 0, 1, 1],[3, 0, 0, 0, 1],[3, 0, 0, 1, 0],[3, 0, 1, 1, 1]],
[[2, 0, 1, 1, 0],[2, 0, 1, 1, 1],[2, 0, 0, 0, 1],[3, 0, 1, 1, 0],[3, 0, 1, 1, 0]],
[[1, 0, 1, 0, 1],[2, 0, 1, 1, 0],[3, 0, 1, 1, 1],[3, 0, 0, 0, 1],[3, 0, 1, 0, 1]],
[[1, 0, 1, 1, 1],[1, 0, 1, 1, 0],[2, 0, 0, 1, 1],[2, 0, 1, 0, 1],[2, 0, 1, 1, 1]],
[[0, 0, 0, 1, 0],[1, 0, 1, 1, 0],[2, 0, 1, 0, 1],[2, 0, 0, 1, 0],[3, 0, 0, 1, 1]]
]
A partition for sets of sizes:  0*3  0*4  9*5
```

I Program to check a partition

We offer a simple program in Python 3 that allows to check easily if a partition is a zero-sum partition. It can be executed in a terminal (or in an online Python environment like https://www.online-python.com/ or https://trinket.io/python), with the three elements of the input: description of the group, sizes of the sets in the partition, and the partition itself, copied-pasted from the annexes.

```
import json

group = json.loads(input())
sizes = json.loads(input())
sets = json.loads(input())
ok = True
for set in sets:
    sums = [0 for pos in range(len(group))]
    for elem in set:
        for pos in range(len(group)):
            sums[pos] = (sums[pos] + elem[pos]) % group[pos]
    if not sums == [0 for i in range(len(group))]:
        ok = False
        break
if ok:
    print("Zero-sum partition")
else:
    print("Not a zero-sum partition")
```