

# A note on removable edges in near-bricks

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An edge  $e$  of a matching covered graph  $G$  is removable if  $G - e$  is also matching covered. Carvalho, Lucchesi, and Murty showed that every brick  $G$  different from  $K_4$  and  $\overline{C}_6$  has at least  $\Delta - 2$  removable edges, where  $\Delta$  is the maximum degree of  $G$ . In this paper, we generalize the result to irreducible near-bricks, where a graph is irreducible if it contains no single ear of length three or more.

**Keywords:** removable edge, perfect matching, near-brick, brick

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## 1 Introduction

All the graphs considered in this paper may have multiple edges, but no loops. We follow Bondy and Murty (2008) for undefined notations and terminologies. Let  $G$  be a graph with the vertex set  $V(G)$  and the edge set  $E(G)$ . We denote by  $\Delta(G)$ , or simply  $\Delta$ , the maximum degree of the vertices of  $G$ . For a subset  $X$  of  $V(G)$ , let  $G[X]$  denote the subgraph of  $G$  induced by  $X$ . A *matching* of a graph is a set of pairwise nonadjacent edges. A *perfect matching* is one which covers every vertex of the graph. A nontrivial connected graph is *matching covered* if each edge lies in a perfect matching of the graph. Clearly, every matching covered graph different from  $K_2$  is 2-connected.

For a nonempty proper subset  $X$  of  $V(G)$ , let  $\partial(X)$  denote the set of all the edges of  $G$  with one end in  $X$  and the other end in  $\overline{X}$ , where  $\overline{X} := V(G) \setminus X$ . The set  $\partial(X)$  is called a *cut* of  $G$ , the sets  $X$  and  $\overline{X}$  its shores. The shore  $X$  of  $\partial(X)$  is *bipartite* if the induced subgraph  $G[X]$  is bipartite. A cut is *trivial* if one of its shores is a singleton, and is *nontrivial* otherwise. We denote by  $G/(X \rightarrow x)$ , or simply  $G/X$ , the graph obtained from  $G$  by shrinking  $X$  to a single vertex  $x$ . Similarly, we denote by  $G/(\overline{X} \rightarrow \overline{x})$ , or simply  $G/\overline{X}$ , the graph obtained from  $G$  by shrinking  $\overline{X}$  to a single vertex  $\overline{x}$ . The two graphs  $G/X$  and  $G/\overline{X}$  are called the two  $\partial(X)$ -*contractions* of  $G$ .

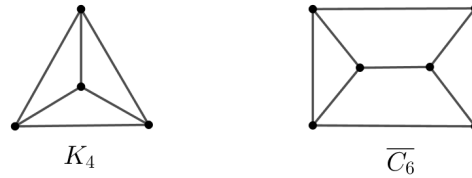
Let  $G$  be a matching covered graph. A cut  $C$  of  $G$  is *tight* if  $|M \cap C| = 1$  for each perfect matching  $M$  of  $G$ . A matching covered graph which is free of nontrivial tight cuts is a *brace* if it is bipartite, and is a *brick* otherwise. If  $G$  has a nontrivial tight cut  $C$ , then each  $C$ -contraction of  $G$  is a matching covered graph that has strictly fewer vertices than  $G$ . Continuing in this way, we can obtain a list of

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matching covered graphs without nontrivial tight cuts, which are bricks and braces. This procedure is known as a *tight cut decomposition* of  $G$ . In general, a matching covered graph may admit several tight cut decompositions. Lovász (1987) showed that any two tight cut decompositions of a matching covered graph yield the same list of bricks and braces (up to multiple edges). This implies that the number of bricks is uniquely determined by  $G$ . Let  $b(G)$  denote the number of the bricks of  $G$ . Note that  $b(G) = 0$  if and only if  $G$  is bipartite.

A graph  $G$  is a *near-brick* if it is a matching covered graph with  $b(G) = 1$ . Clearly, a near-brick is 2-connected and a brick is a near-brick. A *single ear* of a graph is a path of odd length whose internal vertices (if any) all have degree two in this graph. A graph is *irreducible* if it contains no single ear of length three or more. Edmonds et al. (1982) proved that a graph  $G$  is a brick if and only if  $G$  is 3-connected and  $G - x - y$  has a perfect matching for any two distinct vertices  $x, y \in V(G)$ . Therefore, a brick is irreducible. However, a near-brick is not necessarily irreducible. For instance, subdividing an edge of a graph in Figure 1 by inserting two vertices results in a near-brick, which is not irreducible.



**Fig. 1:** The two bricks.

An edge  $e$  of a matching covered graph  $G$  is *removable* if  $G - e$  is also matching covered, and is *nonremovable* otherwise. Clearly, each multiple edge of a matching covered graph is in fact a removable edge. The notion of removable edge is related to ear decompositions of matching covered graphs introduced by Lovász and Plummer. Lovász (1987) showed that every brick distinct from  $K_4$  and  $\overline{C_6}$  has a removable edge, where  $K_4$  and  $\overline{C_6}$  are shown in Figure 1. Carvalho, Lucchesi, and Murty proved the following stronger result.

**Theorem 1.1 (Carvalho et al. (1999))** *Every brick  $G$  different from  $K_4$  and  $\overline{C_6}$  has at least  $\Delta - 2$  removable edges.*

The following theorem is our main result which generalizes the above theorem to irreducible near-bricks.

**Theorem 1.2** *Every irreducible near-brick  $G$  different from  $K_4$  and  $\overline{C_6}$  has at least  $\Delta - 2$  removable edges.*

The paper is organized as follows. In Section 2, we present some basic results. In Section 3, we give a proof of Theorem 1.2.

## 2 Preliminaries

**Lemma 2.1 (Carvalho et al. (1999))** *In a brace on six or more vertices, every edge is removable.*

**Lemma 2.2 (Fabres et al. (2021))** *Every brace on six or more vertices is 3-connected.*

**Lemma 2.3 (Zhang et al. (2022))** *Let  $C$  be a tight cut of a matching covered graph  $G$  and  $e$  an edge of  $G$ . Then  $e$  is removable in  $G$  if and only if  $e$  is removable in each  $C$ -contraction of  $G$  which contains it.*

The following equality reveals an important property of the numbers of bricks of matching covered graphs with respect to tight cuts.

**Lemma 2.4 (Carvalho et al. (2002))** *Let  $G$  be a matching covered graph and  $C$  a tight cut of  $G$ . Let  $G_1$  and  $G_2$  be the two  $C$ -contractions of  $G$ . Then  $b(G) = b(G_1) + b(G_2)$ .*

Using the above lemma, we can easily obtain the following result, also see Carvalho et al. (2002).

**Lemma 2.5 (Carvalho et al. (2002))** *For any tight cut  $C$  of a near-brick  $G$ , precisely one of the shores of  $C$  is bipartite.*

To bisubdivide an edge  $e$  of a graph  $G$  is to replace  $e$  by an odd path with length at least three. The resulting graph is called a *bisubdivision* of  $G$  at the edge  $e$ . Let  $RE(G)$  denote the set of all the removable edges of  $G$ .

**Lemma 2.6** *Let  $G$  be a graph and let  $H$  be a bisubdivision of  $G$  at an edge  $e$ . Suppose that  $H$  is a matching covered graph. Then  $G$  is a matching covered graph with  $b(G) = b(H)$  and  $RE(H) = RE(G) \setminus \{e\}$ .*

**Proof:** Since  $H$  is a bisubdivision of  $G$  at the edge  $e$ ,  $H$  is obtained from  $G$  by replacing  $e$  by an odd path  $P$  with length at least three. We assert that  $G$  is not isomorphic to  $K_2$ . Otherwise,  $H$  is an odd path, contradicting the assumption that  $H$  is a matching covered graph. Let  $e = uv$  and  $X = V(P) \setminus \{v\}$ . Then  $G$  is isomorphic to  $H/X$ . Since  $P - v$  is an even path of  $H$  with all internal vertices of degree 2 in  $H$ , for each perfect matching  $M$  of  $H$ , we have  $|M \cap \partial_H(X)| = 1$ . Then  $\partial_H(X)$  is a tight cut of  $H$ . Since  $H$  is a matching covered graph, so does  $G$ . Since  $G$  is not isomorphic to  $K_2$ ,  $G$  is 2-connected. So  $u$  has at least two neighbours in  $G$ . This implies that the underlying simple graph of  $H/\overline{X}$  is an even cycle. So  $b(H/\overline{X}) = 0$ . By Lemma 2.4, we have  $b(H) = b(G) + b(H/\overline{X}) = b(G)$ .

Now we proceed to show that  $RE(H) = RE(G) \setminus \{e\}$ . Note that each edge of  $P$  is incident with a vertex of degree 2 in  $H$ , and hence is nonremovable in  $H$ . Thus, if  $f \in RE(H)$ , then  $f \in RE(G) \setminus \{e\}$ . By Lemma 2.3, we have  $f \in RE(G) \setminus \{e\}$ . So  $RE(H) \subseteq RE(G) \setminus \{e\}$ . Now assume that  $f \in RE(G) \setminus \{e\}$ . If  $f \notin \partial_H(X)$ , Lemma 2.3 implies that  $f \in RE(H)$ . If  $f \in \partial_H(X)$ , then  $f$  is incident with  $u$  in  $H$ . Since  $f$  is removable in  $G$ , we have  $d_G(u) \geq 3$ . So  $d_H(u) = d_G(u) \geq 3$ . It follows that  $f$  is a multiple edge of  $H/\overline{X}$ , and then is a removable edge of  $H/\overline{X}$ . Again by Lemma 2.3, we have  $f \in RE(H)$ . It follows that  $RE(G) \setminus \{e\} \subseteq RE(H)$ . Consequently,  $RE(H) = RE(G) \setminus \{e\}$ .  $\square$

### 3 Proof of Theorem 1.2

Suppose that  $G$  is an irreducible near-brick different from  $K_4$  and  $\overline{C_6}$ , and  $\Delta = \Delta(G)$ . Then  $b(G) = 1$  and  $|V(G)| \geq 4$ . Moreover,  $G$  is 2-connected and matching covered. So  $\delta(G) \geq 2$ . If  $\Delta < 3$ , then each vertex of  $G$  has degree two. Thus  $G$  is an even cycle. This implies that  $b(G) = 0$ , a contradiction. Therefore,  $\Delta \geq 3$ . We shall show that  $G$  has at least  $\Delta - 2$  removable edges by induction on  $|V(G)| + |E(G)|$ . Now we consider the following two cases according to whether  $G$  has parallel edges or not.

**Case 1.**  $G$  has parallel edges.

Suppose that  $G$  has two parallel edges, say  $e_1, e_2$ , which have common ends. Then  $G - e_1$  is a near-brick, but it has strictly fewer edges than  $G$ . Moreover, we have  $\Delta(G - e_1) \geq \Delta - 1$ . Recall that each multiple edge of a matching covered graph is a removable edge. Then both  $e_1$  and  $e_2$  are removable in  $G$ , that is,  $\{e_1, e_2\} \subseteq RE(G)$ .

*Claim 1.*  $RE(G - e_1) \subseteq RE(G)$ .

Suppose that  $f \in RE(G - e_1)$ . If  $f = e_2$ , then  $f \in RE(G)$ . If  $f \neq e_2$ , then  $G - e_1 - f$  is matching covered, and both  $e_1$  and  $e_2$  are multiple edges of  $G - f$ . Therefore,  $G - f$  is matching covered and then  $f \in RE(G)$ . Claim 1 holds.

If  $G - e_1$  is one of  $K_4$  and  $\overline{C_6}$ , then  $\Delta = 4$  and  $G$  has exactly two removable edges  $e_1$  and  $e_2$ . The result holds. We may thus assume that  $G - e_1$  is neither  $K_4$  nor  $\overline{C_6}$ . If  $G - e_1$  is irreducible, by the induction hypothesis,  $G - e_1$  has at least  $\Delta(G - e_1) - 2$  removable edges, that is,  $|RE(G - e_1)| \geq \Delta(G - e_1) - 2$ . Recall that  $e_1 \in RE(G)$  and  $\Delta(G - e_1) \geq \Delta - 1$ . By Claim 1,

$$|RE(G)| \geq |RE(G - e_1)| + 1 \geq \Delta(G - e_1) - 2 + 1 \geq \Delta - 2.$$

Therefore,  $G$  has at least  $\Delta - 2$  removable edges.

If  $G - e_1$  is not irreducible, since  $G$  is irreducible,  $G - e_1$  has a single ear with length at least 3, which contains  $e_2$ . Let  $P_{e_2}$  be such a maximal single ear, and  $s$  and  $t$  the two ends of  $P_{e_2}$ . Since  $P_{e_2}$  contains  $e_2$  in  $G - e_1$ ,  $e_2$  is incident with a vertex of degree 2 in  $G - e_1$ . This implies that  $e_2 \notin RE(G - e_1)$ . Let  $G'$  be the graph obtained from  $G - e_1 - (V(P_{e_2}) \setminus \{s, t\})$  by adding a new edge  $e$  that connects  $s$  and  $t$ . For each vertex  $x^*$  of  $G'$ , we can see that  $d_{G'}(x^*) = d_{G - e_1}(x^*)$ . In particular, if  $x^* \notin \{s, t\}$ , then  $d_{G'}(x^*) = d_{G - e_1}(x^*) = d_G(x^*)$ . Moreover, we have  $\Delta(G') = \Delta(G - e_1)$ .

*Claim 2.*  $G'$  is a near-brick and  $RE(G - e_1) = RE(G') \setminus \{e\}$ .

Note that  $G - e_1$  is a bisubdivision of  $G'$  at the edge  $e$ . Since  $G - e_1$  is a near-brick, by Lemma 2.6,  $G'$  is a near-brick and  $RE(G - e_1) = RE(G') \setminus \{e\}$ . Claim 2 holds.

*Claim 3.*  $G'$  is irreducible.

Assume that both  $s$  and  $t$  have degree two in  $G'$ . Then both  $s$  and  $t$  have degree two in  $G - e_1$ . Recall that  $G - e_1$  is a near-brick. Then  $G - e_1$  is 2-connected and is not an even cycle. So we have  $st \notin E(G - e_1)$ . Let  $s_1$  be the only vertex in  $N_{G - e_1}(s) \setminus V(P_{e_2})$  and  $t_1$  the only vertex in  $N_{G - e_1}(t) \setminus V(P_{e_2})$ . If  $s_1 = t_1$ , then  $G - e_1 - s_1$  has an even component  $P_{e_2}$ . This implies that  $s_1$  is a cut vertex of  $G - e_1$ , contradicting the fact that  $G - e_1$  is 2-connected. So  $s_1 \neq t_1$ . Then  $P_{e_2} + ss_1 + tt_1$  is a single ear of  $G - e_1$  that contains  $e_2$  and longer than  $P_{e_2}$ , contradicting the maximal of  $P_{e_2}$ . Therefore, at least one of  $s$  and  $t$  have degree three or more in  $G'$ . Assume, without loss of generality, that  $s$  has degree three or more in  $G'$ . If  $t$  has degree three or more in  $G'$ , since  $G$  is irreducible, so does  $G'$ . The claim holds. Now assume that  $t$  has degree two in  $G'$ . If  $N_{G'}(t) = \{s\}$ , then  $st \in E(G - e_1)$ . This implies that  $s$  is a cut vertex of  $G - e_1$ , a contradiction. So  $t$  has exactly two distinct neighbours in  $G'$ . Let  $t'$  be the only vertex in  $N_{G'}(t) \setminus \{s\}$ . Then  $t'$  is the only vertex in  $N_{G - e_1}(t) \setminus V(P_{e_2})$ . If  $t'$  has degree two in  $G - e_1$ , since  $G - e_1$  is 2-connected,  $t'$  has exactly one neighbour other than  $t$ , say  $t''$ , and  $t'' \neq s$ . This implies that  $P_{e_2} + tt't''$  is a single ear of  $G - e_1$  that contains  $e_2$  and longer than  $P_{e_2}$ , contradicting the maximal of  $P_{e_2}$ . So  $t'$  has degree three or more in  $G - e_1$ , and then it has degree three or more in  $G'$ . Since  $s$  has degree three or more in  $G'$  and  $G$  is irreducible,  $G'$  is irreducible. Claim 3 holds.

If  $G'$  is one of  $K_4$  and  $\overline{C_6}$ , then  $\Delta(G - e_1) = \Delta(G') = 3$ . So we have  $\Delta \leq \Delta(G - e_1) + 1 = 4$ . Recall that  $\{e_1, e_2\} \subseteq RE(G)$ . The result holds. Now suppose that  $G'$  is different from  $K_4$  and  $\overline{C_6}$ . By the induction hypothesis,  $G'$  has at least  $\Delta(G') - 2$  removable edges, that is,  $|RE(G')| \geq \Delta(G') - 2$ . By Claim 2, we have  $RE(G - e_1) = RE(G') \setminus \{e\}$ . Then  $|RE(G - e_1)| \geq |RE(G')| - 1 \geq \Delta(G') - 3$ . By Claim 1, we have  $RE(G - e_1) \subseteq RE(G)$ . Recall that  $e_2 \notin RE(G - e_1)$  and  $\{e_1, e_2\} \subseteq RE(G)$ . Then

$$|RE(G)| \geq |RE(G - e_1)| + 2 \geq \Delta(G') - 3 + 2 = \Delta(G - e_1) - 1 \geq \Delta - 2.$$

That is,  $G$  has at least  $\Delta - 2$  removable edges.

**Case 2.**  $G$  is simple.

Note that  $C_4$  and  $K_4$  are the only two simple matching covered graphs with four vertices. Since  $G$  is a near-brick different from  $K_4$  and  $|V(G)| \geq 4$ , we have  $|V(G)| \geq 6$ .

If  $G$  is a brick, by Theorem 1.1, the result holds. So we may assume that  $G$  is not a brick. Since  $b(G) = 1$ ,  $G$  is not a brace and  $G$  has a nontrivial tight cut. Let  $C := \partial(X)$  be a nontrivial tight cut of  $G$ . By Lemma 2.5, we may assume that  $G[X]$  is bipartite, subject to this,  $X$  is minimal. Let  $G_1 = G/(X \rightarrow x)$  and  $G_2 = G/(\overline{X} \rightarrow \overline{x})$ . Then  $G_1$  is matching covered and  $G_2$  is a brace. By Lemma 2.4,  $G_1$  is a near-brick. Assume that  $(B, I)$  is the bipartition of  $G_2$  such that  $\overline{x} \in I$ . Then  $X = B \cup (I \setminus \{\overline{x}\})$ . Let  $u$  be a vertex of  $G$  such that  $d_G(u) = \Delta$ , and write  $\Delta_1 = \Delta(G_1)$ .

First suppose that  $|V(G_2)| \geq 6$ . Then  $|I| \geq 3$ . By Lemma 2.1, each edge of  $G_2$  is removable in  $G_2$ . By Lemma 2.3, each removable edge of  $G_1$  is also a removable edge of  $G$ , that is,  $RE(G_1) \subseteq RE(G)$ . Since  $G_2$  is a brace and  $|V(G_2)| \geq 6$ , by Lemma 2.2, we have  $\delta(G_2) \geq 3$ . Then each vertex in  $X$  has degree three or more in  $G$ , and  $d_{G_1}(x) = d_{G_2}(\overline{x}) \geq 3$ . Since  $G$  is irreducible, so does  $G_1$ . If  $G_1$  is one of  $K_4$  and  $\overline{C_6}$ , then  $|C| = 3$  and each vertex of  $\overline{X}$  has degree three in  $G$ . We may thus assume that  $u \in X$ . We assert that  $|\partial_G(u) \cap C| \leq 2$ . Otherwise,  $u$  is a cut vertex of  $G$ , contradicting the fact that  $G$  is 2-connected. Since every edge of  $G_2$  is removable in  $G_2$ , by Lemma 2.3, each edge of  $G[X]$  is removable in  $G$ . This implies that  $G$  has at least  $|\partial_G(u) \setminus C| \geq \Delta - 2$  removable edges. Now assume that  $G_1$  is different from  $K_4$  and  $\overline{C_6}$ . By the induction hypothesis,  $G_1$  has at least  $\Delta_1 - 2$  removable edges, that is,  $|RE(G_1)| \geq \Delta_1 - 2$ .

If  $u \in \overline{X}$ , then  $\Delta_1 \geq \Delta$ . Since  $RE(G_1) \subseteq RE(G)$ , we have

$$|RE(G)| \geq |RE(G_1)| \geq \Delta_1 - 2 \geq \Delta - 2.$$

So  $G$  has at least  $\Delta - 2$  removable edges. Now assume that  $u \in X$ . Recall that each edge of  $G[X]$  is removable in  $G$ . If  $u \in I \setminus \{\overline{x}\}$ , then each edge incident with  $u$  is removable in  $G$ . This implies that  $G$  has at least  $\Delta$  removable edges. If  $u \in B$ , since  $G$  is simple and  $u$  has at most  $|I| - 1$  neighbours in  $I \setminus \{\overline{x}\}$ , we have  $|C| \geq d_G(u) - (|I| - 1) = \Delta - |I| + 1$ . Then  $\Delta_1 \geq d_{G_1}(x) = |C| \geq \Delta - |I| + 1$ . Note that each edge which is incident with a vertex in  $I \setminus \{\overline{x}\}$  is removable in  $G$ . Since  $RE(G_1) \subseteq RE(G)$  and  $\delta(G_2) \geq 3$ , we have

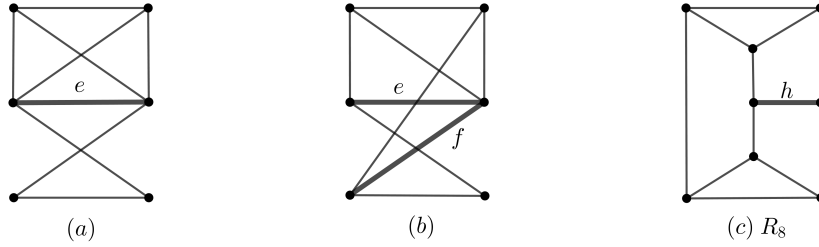
$$|RE(G)| \geq |RE(G_1)| + 3|I \setminus \{\overline{x}\}| \geq \Delta_1 - 2 + 3(|I| - 1) \geq \Delta - |I| + 1 - 5 + 3|I| = \Delta - 4 + 2|I| \geq \Delta + 2.$$

Therefore,  $G$  has at least  $\Delta + 2$  removable edges.

Now suppose that  $|V(G_2)| = 4$ . Then  $|B| = 2$  and  $|I \setminus \{\overline{x}\}| = 1$ . Moreover, the only vertex in  $I \setminus \{\overline{x}\}$  has degree two in  $G$  because  $G$  is simple. Since  $G$  is irreducible, each vertex in  $B$  has degree three or more in  $G$ . Thus, we have  $d_{G_1}(x) = d_{G_2}(\overline{x}) = |C| \geq 4$ , and each edge of  $C$  is a multiple edge of  $G_2$ .

Then each edge of  $C$  is a removable edge of  $G_2$ . By Lemma 2.3, we have  $RE(G_1) \subseteq RE(G)$ . Since  $d_{G_1}(x) \geq 4$  and  $G$  is irreducible,  $G_1$  is irreducible and is different from  $K_4$  and  $\overline{C_6}$ . By the induction hypothesis,  $G_1$  has at least  $\Delta_1 - 2$  removable edges, that is,  $|RE(G_1)| \geq \Delta_1 - 2$ . If  $\Delta_1 \geq \Delta$ , since  $RE(G_1) \subseteq RE(G)$ , we have  $|RE(G)| \geq |RE(G_1)| \geq \Delta_1 - 2 \geq \Delta - 2$ . Then  $G$  has at least  $\Delta - 2$  removable edges. To complete the proof, we now show that  $\Delta_1 \geq \Delta$ . Clearly, it is true when  $u \in \overline{X}$ . We may assume that  $u \in X$ . Then  $u \in B$  and  $\Delta_1 \geq d_{G_1}(x) = d_{G_2}(\overline{x}) \geq d_G(u) - 1 + 2 = \Delta + 1$ . Theorem 1.2 holds.  $\square$

**Remark.** The condition of Theorem 1.2 that the graph is irreducible is necessary. For instance, the graph in Figure 2(a) is a near-brick with maximum degree four but not irreducible, and has exactly one removable edge  $e$ . Furthermore, the lower bound of Theorem 1.2 is sharp. The graph shown in Figure 2(b) is an irreducible near-brick with maximum degree four and has exactly two removable edges  $e$  and  $f$ ; the graph  $R_8$  shown in Figure 2(c) is a cubic brick with exactly one removable edge  $h$ .



**Fig. 2:** The three near-bricks.

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